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Fractal geometry in basic education: a systematic review of pedagogical approaches and learning strategies

ARTICLE

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Abstract
This study aims to map and analyze research that proposes didactic activities focused on Fractal Geometry in Mathematics education, identifying methodologies, resources used, and their impact on the learning process. Through a systematic literature review (SLR), nine dissertations were selected from CAPES journals and the PROFMAT repository. The results show a growing presence of Fractal Geometry in high school education, with frequent use of software such as GeoGebra and manipulative materials that enhance visualization and experimentation of fractal concepts. Pedagogical approaches such as constructivism and interactionism are also observed, promoting active student learning. It is concluded that the inclusion of Fractal Geometry can contribute to student motivation and engagement, although challenges remain, such as the need for greater integration with other areas of knowledge and the inclusion of programming education as a complementary tool.
Keywords: High School. Teaching-learning. Fractal Geometry. Mathematics.

Geometria fractal no ensino médio: uma revisão sistemática de abordagens pedagógicas e estratégias de aprendizagem

Resumo
Este estudo objetiva mapear e analisar pesquisas que propõem atividades didáticas voltadas à Geometria Fractal no ensino de Matemática, identificando metodologias, recursos e impactos no processo de aprendizagem. Por meio de uma revisão sistemática da literatura (RSL), foram selecionadas nove dissertações disponíveis nos periódicos da CAPES e no repositório do PROFMAT. Os resultados revelam crescente presença da Geometria Fractal no Ensino Médio, com uso frequente de softwares como o GeoGebra e materiais manipulativos, favorecendo a visualização e experimentação dos conceitos. Observa-se ainda a adoção de abordagens pedagógicas como o construtivismo e o interacionismo, que promovem a aprendizagem ativa. Conclui-se que a inserção da Geometria Fractal pode contribuir para a motivação e o engajamento dos estudantes, embora persistam desafios, como a integração com outras áreas do conhecimento e a inclusão do ensino de programação como ferramenta complementar.
Palavras-chave: Ensino Médio. Ensino-aprendizagem. Geometria Fractal. Matemática.

1 Introduction

Over the years, mathematics education has faced challenges and opportunities stemming from scientific and technological developments. Fractal Geometry, formalized by Benoît Mandelbrot in the 1970s, studies self-referential geometric patterns characterized by self-similarity across different scales and infinite complexity generated by simple iterative rules. This mathematical approach finds applications not only within Mathematics itself, but also in fields such as Physics (the study of chaotic systems), Biology (modeling cellular growth and natural patterns), Geography (analysis of coastal formations and watersheds), and Computer Science (image compression and procedural generation algorithms). These interdisciplinary connections make fractals a powerful educational tool, enabling projects and activities that explore mathematical modeling and digital visualization. Studies such as those by Barbosa (2005) and Mossulin and Medeiros (2023) demonstrate the feasibility of using Fractal Geometry in hands-on classroom activities, fostering greater student engagement through computational tools like GeoGebra software and active methodologies based on experimentation.

Although fractal geometry is still considered a relatively new field, its relevance in education has seen significant growth. This trend is clearly reflected in official curricular documents, notably the Base Nacional Comum Curricular (BNCC – National Common Curricular Base), which highlights fractals as fundamental instruments for understanding both human-made creations and natural patterns (Brasil, 2017).

The introduction of fractal geometry into the educational curriculum framework represents not only a methodological innovation, but also an opportunity to reestablish students' connection with mathematical concepts through practical and contextual applications. The exploration of fractal patterns allows students to deepen their understanding of complex phenomena, ranging from natural occurrences such as clouds and mountainous landscapes to artificial constructs, exemplified by urban infrastructure networks and computational systems.

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However, the inclusion of fractal geometry in the teaching process requires a well-planned and structured pedagogical approach, taking into account the various possibilities and teaching strategies. While this approach has the potential to engage students and broaden their mathematical understanding, its effectiveness depends on factors such as classroom profile, students' interests, and the teacher's mediation. Thus, didactic strategies based on experimentation, mathematical modeling, and the use of technology can foster learning, but do not deterministically guarantee content assimilation for all students.

Within this context, the present systematic literature review seeks to map studies that advocate for didactic activities centered on the exploration of Fractal Geometry in Mathematics education. The scope encompasses an analysis of the existing literature with the aim of identifying effective pedagogical practices, assessing their impact on student engagement and performance, and exploring potential implications for students' mathematical and cognitive development.

Additionally, this review aims to situate fractal geometry within the broader landscape of contemporary mathematics education, highlighting its intersections with other fields of knowledge and outlining promising perspectives for future research. Through this effort, it is hoped that a substantial contribution can be made to the advancement of the field and to the promotion of an interdisciplinary, student-centered educational approach.

2 Methodology

The purpose of this systematic literature review lies in the apprehension and evidencing of the underlying logic of a particular set of documents, with an emphasis on the replicability of research within the academic community. This approach draws upon the contributions outlined by Galvão and Ricarte (2019). The present study is based on an exhaustive review of information related to a specific topic, aiming to address an investigative question.

Following the methodological framework proposed by Mossulin and Medeiros (2023), this RSL adopted a structured protocol, encompassing the following components:

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i) guiding research question;
ii) specific research questions;
iii) development of search strings;
iv) inclusion and exclusion criteria;
v) delimitation of the document search scope;
vi) detailed presentation of the results;
vii) subsequent analysis.

With regard to the procedures for conducting this RSL, the process began with the screening of titles, followed by the review of abstracts from the previously selected studies. Subsequently, a thorough examination was carried out on the works directly aligned with the objectives of this investigation. Additionally, it is important to highlight that the time frame acted as a guiding criterion, given the need to gather information relevant to topics of prominent interest that have emerged over the past five years.

To identify research related to Fractal Geometry and its pedagogical approaches in the context of secondary education (Ensino Médio), the following descriptors were considered: “Fractal Geometry” and “teaching and learning.” Prioritizing *stricto sensu* graduate programs and targeting works written in Portuguese, the selected databases included the “Portal de Periódicos da CAPES” (CAPES Journal Portal), accessed via CAFe, and the repository of the Mestrado Profissional em Matemática em Rede Nacional (PROFMAT – National Network Professional Master's in Mathematics).

The research considered the time frame from 2018 to 2023, resulting in the identification of eighty studies. Among these, only dissertations were deemed relevant for the purposes of this article, as the aforementioned portal did not yield applicable results for the selected period. In the CAPES Journal Portal, four studies related to Fractal Geometry in education were identified. However, these were excluded for not meeting the established inclusion criteria, such as lack of focus on secondary education, absence of applied methodology descriptions, or being theoretical reviews without specific didactic proposals. On the other hand, a preliminary search in PROFMAT led to the identification of 21 studies

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related to the theme. However, only nine of these were considered aligned with the objectives of the present research.

A summary of the searches conducted in the selected databases is presented in Box 1.

Box 1 - Result global from search from RSL

Platforms	Papers found on all the themes	Included papers analyzed in full
CAPES	80	0
PROFMAT	21	9
Total	**101**	**9**

Source: Prepared by the authors (2024)

Thus, through the analysis of the abstracts of the selected studies, nine works were identified, which serve as the foundation for the analyses presented in this article. Box 2 details the inclusion and exclusion criteria adopted for the previously selected studies, in accordance with the established protocol.

Box 2 - Criteria for inclusion and exclusion

Criteria from Inclusion	Criteria of Exclusion
1. Complete works on the subject of this research 2. Dissertations 3. Geometry fractal 4. Geometry fractal - teaching and learning 5. Geometry fractal - Basic education 6. Works in Portuguese	1. Duplicate works 2. Works on other languages 3. Papers summarized 4. Papers excluded for not indicating keywords in the title, abstract or the relationship with the approach of fractals in education basic

Source: Prepared by the authors (2024).

Box 3 lists the studies selected after applying the inclusion and exclusion criteria, each identified using the format “En,” where *n* represents the corresponding sequential number in the box. The table presents details such as the title, subject, and type of work

("D" for dissertation, "T" for thesis, and "A" for article), the authors' names, year of publication, and the source repository.

Box 3 - References and result global from search from RSL

ID	Title	Type	Year	Authors	Website/ Repository
E1	Fractal Geometry for teaching various math topics in high school	D	2022	Soraia Perez de Souza	PROFMAT – UNESP
E2	Fractal geometry in the teaching-learning-assessment process of geometric probability	D	2021	Sandro Luiz Rosa Able	PROFMAT – UTFPR
E3	Mathematical modeling, fractal geometry and geogebra: proposed activities for teaching high school mathematical content	D	2021	Márcio André Santa Brígida Lima	PROFMAT – UFPA
E4	Fractal geometry and activities for teaching mathematics: fractal steps and Menger's sponge	D	2020	Marcus Vinícius Oliveira Lopes da Silva	PROFMAT – UFBA
E5	The mobilization of Fractal Geometry as a tool for the study of probability in high school	D	2020	Diego de Oliveira Cosim	PROFMAT – UEMS
E6	A proposal for approaching Fractal Geometry in basic education	D	2019	Marcelo Correia Lisboa	PROFMAT – UFT
E7	Fractal Geometry and the contextualization of content in elementary school: a classroom experience	D	2019	Daniel Possetti	PROFMAT – UFMS
E8	Fractal Geometry in secondary education: theory and practice of Fractal Geometry in secondary education	D	2019	Sérgio Nóbrega de Oliveira	PROFMAT – UNIRIO
E9	Getting to know Fractal Geometry from basic concepts of plane Euclidean geometry	D	2019	Marcos Antonio Lima	PROFMAT – UFSC

Source: Prepared by the authors (2024)

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3 Results and Discussion

To support the present analysis of this systematic literature review (RSL), the following guiding question was formulated: *In what ways is Fractal Geometry approached within the context of secondary education, with particular emphasis on strategies inherent to the teaching and learning process?* To address this question, a set of specific research inquiries is proposed and enumerated in Box 4, labeled as “Qn,” where “n” represents the corresponding sequential number.

Box 4 - Research questions

ID	Question
Q1	What are the main objectives of this study?
Q2	How were the activities described in the study designed and conducted?
Q3	What results have been achieved?
Q4	Which pedagogical theories are used?
Q5	What aspects of mathematical knowledge are explored in the study?
Q6	Who is the target audience identified in the study?
Q7	What methods were used in the study to promote teaching and learning among the students?
Q8	What techniques or technologies were used to promote teaching and learning among the students?

Source: Prepared by the authors (2024)

Questions Q7 and Q8 were formulated based on the analysis of the description provided in Q2, aiming to gain a more detailed understanding of the methods, techniques, and technologies employed in the studies. Box 5, as referenced in Q1, presents a mapping related to the objectives pursued in the research works.

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Box 5 - Primary objectives analyzed

Q1: What are the main objectives of this study?		
Code	**Authors**	**Objectives**
E1	Soraia Perez de Souza	The proposal outlined aims to incorporate Fractal Geometry as a motivating agent in the teaching of Euclidean geometric quantities, namely areas and perimeters. In the same vein, with a view to enriching the teaching of mathematics, an approach is envisioned which, by incorporating technologies, aims to catalyze the learning process. It also outlines the design and implementation of activities using the GeoGebra computer platform. These pedagogical initiatives are carried out at secondary school level, with the pressing aim of probing and solidifying mathematical concepts. This objective encompasses a wide range of topics, such as geometric progression, geometry, trigonometry, logarithms and notions of limits, among other relevant subjects.
E2	Sandro Luiz Rosa Able	The aim is to map the works that deal with the subject of Geometric Probability, identifying their approaches, the resources used and the activities proposed.
E3	Márcio André Santa Brígida Lima	To develop and present five Mathematical Modeling activities with challenges originating from fractals, investigated using GeoGebra, as suggestions for teaching mathematical concepts in High School.
E4	Marcus Vinícius Oliveira Lopes da Silva	To develop, in a direct and indirect way, some notions of Fractal Geometry, where the whole process of building the models is clearly explained by means of figures.
E5	Diego de Oliveira Cosim	To investigate how ideas related to Fractal Geometry can be applied to the study of probability in secondary school.
E6	Marcelo Correia Lisboa	A proposal concerning the inclusion of Fractal Geometry in Basic Education is based on the adoption of strategies that combine fractal components with the mathematical content inherent in the teaching cycle corresponding to High School. The primary intention is to instill or improve certain conceptions. The underlying aim of this initiative is to insert this innovative geometric approach into the context of secondary education, endorsing it as a support for the pedagogical process focused on mathematical content. In this way, it is hoped that this approach will catalyze the acquisition of mathematical knowledge, giving it an added axiological charge for students.
E7	Daniel Possetti	This research aims to examine the impact and effectiveness of incorporating Fractal Geometry into the contextualization of curricular components pertinent to secondary education. The primary aim is to scrutinize how this approach reverberates on the conceptual apprehension of students and favors the consolidation of pedagogical methodologies entangled in the context of the school environment.

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E8	Sérgio Nóbrega de Oliveira	To present a proposal for activities that prove to be supportive in contexts that are usually permeated with complexity when subjected to conventional methods of instruction in an educational environment, as well as being discernible from the scope of reality. This proposal, therefore, aims to highlight the ubiquity of fractals in different domains, namely in the scientific, cultural and technological spheres, thus providing the possibility of a contextualized introduction to Fractal Geometry in the context of students' daily experiences.
E9	Marcos Antonio Lima	Familiarize yourself with Fractal Geometry based on fundamental concepts of Plane Euclidean Geometry.

Source: Prepared by the authors (2024)

In Question Q1, it is observed that in studies E1 and E4, the proposed approach aims to familiarize students with Fractal Geometry through elementary concepts of Euclidean Plane Geometry, advocating the use of fractal forms as a driving force for the exploration and application of fundamental geometric content.

In studies E2, E6, and E8, the authors reflect on the general characteristics of fractals and their degree of integration into technological contexts. The scope of these investigations is to encourage students to recognize the relevance of studying fractals and, through their construction, to identify their attributes and applications in everyday life.

With regard to studies E3, E5, and E7, a series of activities is outlined for teaching probability and geometric progressions, using the construction of fractals within the environment provided by the GeoGebra software.

In study E9, the focus is on investigating how concepts related to Fractal Geometry can be effectively incorporated into the teaching of probability in secondary education. Furthermore, the study suggests that this new geometric perspective be integrated into the secondary school curriculum as a facilitating tool in the instruction of mathematical concepts, aiming to strengthen mathematical understanding and make it more meaningful for students.

Similar to Box 5, Box 6 presents a methodological description related to the development of the activities (Q2), including a column indicating authorship to allow for more precise identification of the research conducted.

Box 6 - Description methodological cited in study

Q2: How were the activities described in the study designed and conducted?		
ID	**Author**	**Description of Methodologies**
E1	Soraia Perez de Souza	It is submitted for consideration the development of computer activities, aimed at the core of the teaching environment corresponding to secondary education. These activities are designed to scrutinize mathematical concepts of various kinds, including Geometric Progression, Geometry, Trigonometry, Logarithms, and notions related to limits, among other topics.
E2	Sandro Luiz Rosa Able	The theory of the three pedagogical moments by Delizoicov and Angotti (1990) was employed as the methodological reference for the development of the instructional sequence.
E3	Márcio André Santa Brígida Lima	In this context, the use of GeoGebra software aimed to catalyze visualization and experimentation, resulting in the creation of mathematical models that reflect the geometry underlying fractals. Along this trajectory, several mathematical principles were defined.
E4	Marcus Vinícius Oliveira Lopes da Silva	Didactic activities were designed to be carried out during mathematics classes at the secondary education level. The process began with an in-depth analysis of Fractal Geometry, including its historical and conceptual foundations. Subsequently, its applications in other scientific domains were thoroughly explored, as well as its use in mathematics teaching activities.
E5	Diego de Oliveira Cosim	The research was conducted in two distinct phases: an initial phase of bibliographic research, followed by a second phase involving fieldwork.
E6	Marcelo Correia Lisboa	This proposal entails the implementation of actions that incorporate elements of Fractal Geometry into secondary-level mathematics content, with the aim of introducing or consolidating mathematical concepts.
E7	Daniel Possetti	To assess the potential of these activities as a didactic resource, a study was conducted involving the implementation of activities on Fractal Geometry with ninth-grade students at a school in Biriguí-SP, in 2018.
E8	Sérgio Nóbrega de Oliveira	Various research sources were used throughout the study, with particular emphasis on the book *Descobrindo a Geometria Fractal* (*Discovering Fractal Geometry*) by Professor Ruy Madsen Barbosa, which served as the primary reference.
E9	Marcos Antonio Lima	An intrinsic approach to these concepts was adopted, including some of their axioms, thereby providing students—especially those in secondary education—with a deeper understanding for the construction of fractals.

Source: Prepared by the authors (2024)

Box 7 contains the results identified in the selected studies. It was observed that all the texts present work proposals that have already been implemented, which makes them applicable to the objective of the research question.

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Box 7 – Main methodologies and tools cited in the study

Q3: What results were achieved? Were any alternatives or additional suggestions identified?		
ID	**Author**	**Description of Methodologies and Tools**
E1	Soraia Perez de Souza	GeoGebra software was used to develop activities aimed at the secondary classroom environment, with the objective of promoting the development of mathematical concepts.
E2	Sandro Luiz Rosa Able	The theory of the Three Pedagogical Moments by Delizoicov and Angotti (1990) was employed as a methodological tool in the design of the instructional sequence. The activities were conceived considering the use of manipulable teaching materials for fractal construction.
E3	Marcio Andre Santa Brigida Lima	GeoGebra software was utilized to assist visualization and experimentation, resulting in the creation of mathematical models observed in fractal geometry.
E4	Marcus Vinícius Oliveira Lopes da Silva	Didactic activities were designed for implementation in mathematics classes, directed at the final cycle of elementary education and secondary education. Additionally, a historical and conceptual analysis of geometry in interdisciplinary contexts was conducted, with particular focus on the geometric properties underlying fractal structures.
E5	Diego de Oliveira Cosim	The investigation unfolded in two clearly delineated stages: a preliminary bibliographic research phase preceding a subsequent empirical research phase. In the preliminary phase, the conceptual exposition and exemplification of the applicability of Fractal Geometry were meticulously presented, addressing the distinctive characteristics of certain types of regular fractals in an effort to discern the peculiar properties inherent to fractal dimension. In the subsequent stage, activities related to Fractal Geometry were implemented with a class of 23 students from the third year of secondary education, aiming to establish calculations pertinent to geometric probability.
E6	Marcelo Correia Lisboa	Through bibliographic and exploratory research, based primarily on works by authors such as Boyer (1996), Valente (1999), Pavanello (1993), Barbosa (2005), Janos (2008), and Smole and Diniz (2016), the historical trajectory was traced.
E7	Daniel Possetti	A bibliographic research was conducted, and activities addressing Fractal Geometry were carried out with ninth-grade students in 2018.
E8	Sérgio Nóbrega de Oliveira	The work was mainly based on the book *Descobrindo a Geometria Fractal* (*Discovering Fractal Geometry*) by Professor Ruy Madsen Barbosa, which served as the central reference source.
E9	Marcos Antonio Lima	The initial concepts of Fractal Geometry were explored starting from the fundamentals of Euclidean Plane Geometry. An intrinsic approach to these concepts was undertaken, including the exploration of some of their axioms.

Source: Prepared by the authors (2024)

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When considering the outcomes derived from the approach to Q3, it is possible to discern a plurality of distinct scenarios. The works designated as E1 and E9 are predominantly focused on bibliographic review, whereas E5 and E7 employed an approach supported by the use of GeoGebra software, aiming to facilitate visualization and experimentation, resulting in the development of mathematical models that capture the essence of Fractal Geometry.

From another perspective, studies identified as E2 and E4 incorporated field research methods into their methodology, while E6 adopted the theory of the Three Pedagogical Moments, as outlined by Delizoicov and Angotti (1990), as a methodological tool for structuring the pedagogical sequence. Similarly, E8 was based on historical and conceptual investigations within the scope of geometry, exploring its interdisciplinary connections.

The inquiry into fractal structures was systematically examined, with particular emphasis on the primary source E3, among other referenced sources. Notably, the adoption of the work *Descobrindo a Geometria Fractal* (*Discovering Fractal Geometry*) by Ruy Madsen Barbosa stands out as a fundamental source in this process of conceptual analysis and construction.

Box 8 – Pedagogical Theories

Q4: What pedagogical theories are employed?		
ID	**Pedagogical Theory**	**Description**
E3	Person-Centered Learning (Carl Rogers)	The pedagogical approach prioritizes the role of the teacher as a facilitator of the educational journey, rather than a mere transmitter of knowledge. In this new paradigm, the teacher acts as a mentor, guiding students in understanding life as a process of constant transformation. In this way, learners are encouraged to take initiative in exploring their own cognitive repertoire, recognizing their ongoing evolution as individuals (Rogers, 1969).

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E1, E8	Socio-Interactionism (Vygotsky)	The development and acquisition of knowledge occur through the individual's interactions with peers. Thus, psychological development moves from the interpsychological domain, related to interpersonal relationships, to the intrapsychological domain, referring to the internalized processes within the subject's mental sphere (Vygotsky, 1984).
E4	Constructionism (Papert)	The proposed instructional method aims to achieve the peak of learning with minimal interference in the educational process. Its theoretical foundation is based on the premise that children thrive more effectively when they uncover, by themselves, the specific knowledge they need, as advocated by Papert (2008).
E2, E5	Constructivism (Piaget)	The approach fosters the active engagement of the student in their own educational journey through methodologies that include experimentation, collective inquiry, encouragement of questioning, and enhancement of logical thinking, among other strategies (Piaget, 1973).
E6	Theory of the Three Pedagogical Moments (Delizoicov and Angotti)	The process begins with problematization, followed by the organization and application of knowledge (Delizoicov and Angotti, 1990).
E9	Theory of Semiotic Representation Registers (Raymond Duval)	It holds that external representations serve as tools to facilitate the construction of learning (Durval, 1993).
E7	Van Hiele Theory	The model proposes that students progress through a sequence of levels of understanding of concepts as they develop their skills in geometry (Van Hiele, 1986).

Source: Prepared by the authors (2024)

The issue raised in Q4 prompts an analysis of the underlying pedagogical theoretical foundations in the scrutinized studies. It is observed that study E3 is grounded in the doctrine of Person-Centered Learning, derived from the perspective of Carl Rogers. In turn, E1 and E8 are rooted in the Socio-Interactionist framework, as outlined by Vygotsky. Constructionism, according to the principles put forth by Papert, emerges in study E4. In studies E2, E5, and E7, there is a clear proposition of active student participation in their own educational journey, through experimentation, collective inquiry, encouragement of questioning, and the development of reasoning—hallmarks of Piaget's constructivist approach.

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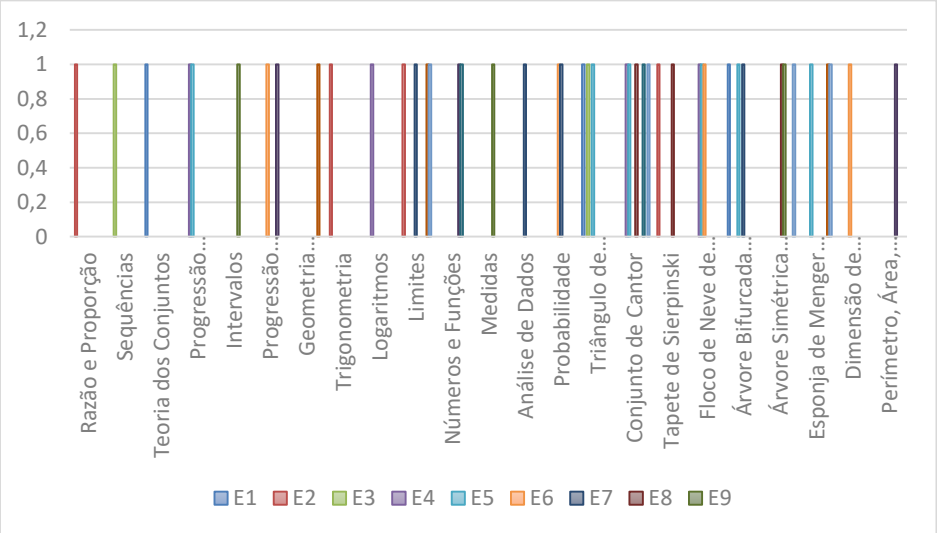
Study E6 delves into the Theory of the Three Pedagogical Moments, conceived by Delizoicov and Angotti (1990), while E9 adopts the framework of the Theory of Semiotic Representation Registers, developed by Raymond Duval.

Figure 1 presents the distribution of mathematical content related to Fractal Geometry found in the analyzed studies. This is a bar chart, in which the x-axis represents the different topics addressed, and the y-axis indicates the frequency with which these topics appear in the research. It is observed that the Cantor set is the most frequently cited fractal, appearing in five instances, followed by the Sierpinski triangle, present in four studies. Other fractals—such as the bifurcated tree, the Pythagorean tree, Koch's snowflake, and the Menger sponge—appear in at least three instances each.

In addition to fractals, complementary mathematical concepts were also identified. Geometric progression is mentioned in six studies, limits appear four times, and probability is cited on two occasions. The Hausdorff dimension, as well as calculations involving fractal perimeter, area, and volume, are each mentioned at least once. This overview suggests that, although the studies on Fractal Geometry prioritize certain specific fractals, there is also a clear effort to connect these elements to fundamental mathematical content.

Figure 1 – Mathematical Content Addressed in the Studies



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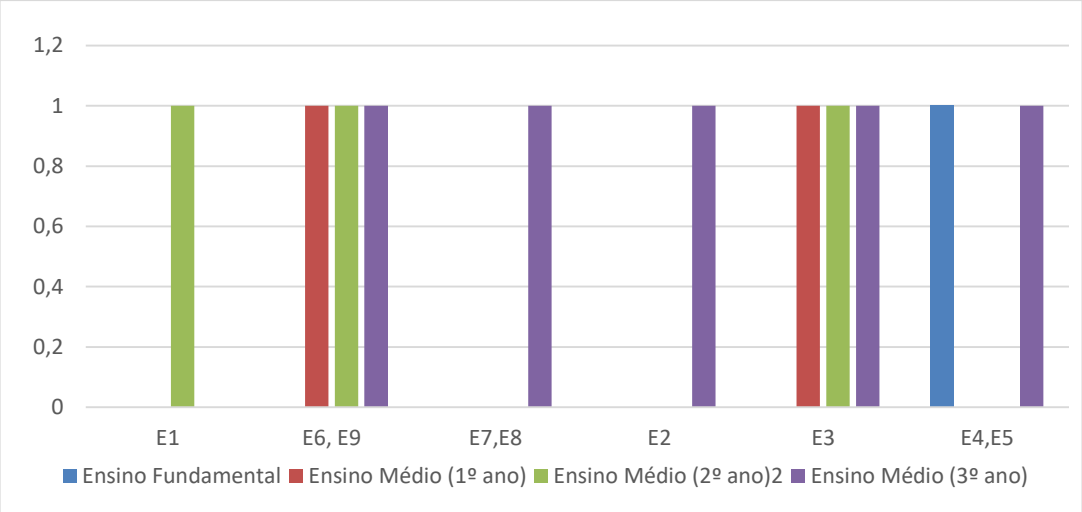
The inquiry posed in Q5 focuses on the mathematical content that underpins the investigations, with particular attention to activities related to Fractal Geometry as they are integrated into the educational curriculum. Regarding the target audience, the analysis is summarized in Figure 2, which shows that, among the nine scrutinized studies, only one addresses elements from the educational cycle known as Elementary School. Consequently, the remaining eight studies focus on the instruction of students enrolled in the subsequent cycle, that is, Upper Secondary Education.

Figure 2 employs a pie chart to represent the distribution of the target audience across the analyzed studies. In terms of percentage, it is observed that only one of the nine studies concentrates on Elementary School education, while the other eight are dedicated to Upper Secondary Education. Among these, study E4 is the only one that directly explores the context of Final Years of Elementary School.

Figure 2 – Target Audience of the Studies



Source: Prepared by the authors (2024)

In Upper Secondary Education, the studies are distributed according to the students' grade levels. Studies E6, E9, and E3 encompass students from the first, second, and third years, respectively, while E7, E8, E2, and E5 focus exclusively on students in

their final year. This data suggests a predominance of research targeting students in more advanced stages of schooling, possibly due to the complexity of concepts associated with Fractal Geometry.

Regarding Q6, the profile of the audience targeted in the studies is outlined. Only study E4 focused directly on the context of Final Years of Elementary School, whereas the remaining studies were directed toward Upper Secondary Education. In particular, studies E6, E9, and E3 included students from the first, second, and third years of Upper Secondary Education, while studies E7, E8, E2, and E5 were limited to students in the final year.

Concerning the methodology adopted in the analyzed studies, Box 7 reveals that in five of them, both theoretical and practical workshops were implemented. In the remaining four studies, the focus was on proposing workshops that had not yet been carried out.

Box 9 – Methods used in the study

Q7: What methods were used in the study to promote teaching and learning among students?	
Studies	**Type of Work**
E2, E3, E5 e E7	Theoretical and Practical Workshops
E1, E4, E6, E8 e E9	Development of Unimplemented Workshop Proposals.

Source: Prepared by the authors (2024)

As in Box 8, Box 9 presents details regarding the techniques and technologies employed in the studies, as outlined in question Q2 and evidenced in question Q8. This box notably highlights the incorporation of a variety of didactic resources, encompassing not only manipulable materials but also the application of the GeoGebra software, which facilitates the establishment of geometric relationships.

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Box 10 – Regarding the techniques/technologies used in the study

Q8: What techniques or technologies were employed to promote teaching and learning among students?	
Studies	**Techniques/Technologies Used**
E3	Rulers and compass
E2, E4 e E8	Manipulable materials
E7	Origami
E5 e E7	Cards
E4 e E6	Handicrafts
E3, E5 e E7	Software (GeoGebra)
E5 e E7	Photography

Source: Prepared by the authors (2024)

In five of the nine studies, workshops were designed and conducted during the students' regular or supplementary school hours, with GeoGebra software employed in three of them (E3, E5, and E7). Additionally, several of these studies incorporated the use of manipulable materials and the application of handicrafts as part of their methodologies.

Among the nine studies analyzed, all were characterized as dissertations available in the PROFMAT repository, since it was not possible to identify dissertations corresponding to the studied period in the CAPES Periodicals Portal. Based on the outcomes presented in the research, the effectiveness of using Fractal Geometry in its role is evidenced, promoting exploration of its relationship with Euclidean geometry and other mathematical elements. According to Mossulin and Medeiros (2023), the investigation of fractals enables students to reach a higher level of motivation, as well as to refine their aesthetic perception. However, it is important to emphasize that certain more abstract concepts, such as fractal dimension, require a longer intervention period and may demand a solid foundation in mathematics, which varies according to different educational levels.

It becomes apparent that the use of a broad range of mathematical content can play a facilitating role in the teaching-learning process related to Fractal Geometry, notably

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due to its alignment with the mathematical competencies and skills advocated by the BNCC. The authors examined showed particular care in integrating pedagogical theories into their approaches, encompassing currents such as constructivism and interactionism, as well as Rogers's theory (1969), which emphasizes student-centered learning, and the theory of semiotic representation registers developed by Duval (1993).

Due to the intrinsic accessibility of the Fractal Geometry approach, there is a notable preference for fractals such as the Cantor set and the Sierpinski triangle, as well as the Peano curve and the Hilbert curve, among others. It is worth highlighting the increasing use of technological resources, notably GeoGebra software, combined with manipulable materials and elements related to handicrafts. Nonetheless, a vast array of possibilities remains to be explored within this theme, including the application of apps and games, programming via tools such as Scratch, and fractal construction in environments designed for the teaching and learning process.

4 Final considerations

This systematic review enabled the mapping of various pedagogical approaches that explore Fractal Geometry in Upper Secondary Education, highlighting its relevance for mathematics teaching. The analyzed studies demonstrated that the use of active methodologies, especially those incorporating technological resources such as GeoGebra, enhances the visualization and understanding of fractal concepts, making learning more dynamic and accessible.

Despite progress in the use of computational tools, a gap was identified regarding the integration of programming instruction within the context of Fractal Geometry. Few of the reviewed works proposed activities involving programming languages for fractal modeling, suggesting an opportunity for future research. The use of environments such as Scratch and Python could offer new possibilities for students to explore fractal creation and manipulation interactively and investigatively.

Furthermore, interdisciplinarity emerged as a relevant aspect, given that Fractal Geometry has applications in various fields of knowledge. However, most of the analyzed studies remain confined to mathematics, which underscores the need for further investigations promoting connections with disciplines such as physics, biology, and computer science.

Finally, it is concluded that the incorporation of Fractal Geometry into Upper Secondary Education holds great potential to enrich mathematics teaching, provided it is accompanied by well-planned strategies adapted to students' realities. Future studies may deepen investigations into new methodologies, teacher training, and the impact of programming instruction as a complementary resource in fractal learning.

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