Instructional design of Mathematics and Language Integrated Learning (MLIL) for future Mathematics teachers

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Abstract
This article focuses on the problem of Mathematics and English Integrated Learning by future Mathematics teachers. Our literature review on the problem shows a variety of aspects that scientists consider when studying the Mathematics and English Integrated Learning model. However, previous researches did not consider the problem of appropriate learning materials for Mathematics and English Integrated Learning in detail. The paper shows the results of two conducted surveys of students majoring in Mathematics and Secondary Education (Mathematics) in Ukraine (82 and 32 participants, respectively). They helped us to make a conclusion about: the importance of the integrated learning for both mathematical and language students’ competence; the importance of syllabus elements reflecting both the mathematical and linguistic components of the goals, objectives, and the expected learning outcomes; the necessity of the Mathematics and English Integrated Learning content to be consistent with the current needs of students; the necessity of using different means of scaffolding, in particular the opportunities of information and communication technologies.

Keywords
content and language integrated learning; Mathematics and Language Integrated Learning; educational support; learning materials; future Mathematics teachers.

Resumo
Este artigo enfoca o problema da Aprendizagem Integrada de Matemática e Inglês por futuros professores de Matemática. Esta revisão de literatura sobre o problema mostra uma variedade de aspectos que os cientistas consideram ao estudar o referido modelo. No entanto, pesquisas anteriores não consideraram em detalhes o problema de materiais de aprendizagem apropriados para a Aprendizagem Integrada de Matemática e Inglês. O artigo mostra os resultados de duas pesquisas realizadas com estudantes de Matemática e Ensino Secundário (Matemática) na Ucrânia (82 e 32 participantes, respectivamente), os quais ajudaram a tirar uma conclusão sobre: a importância da aprendizagem integrada para a competência matemática e de idiomas dos alunos; a importância dos elementos do
programa que refletem os componentes matemáticos e linguísticos das metas, objetivos e resultados de aprendizagem esperados; a necessidade do conteúdo de Aprendizagem Integrada de Matemática e Inglês ser consistente com as necessidades atuais dos alunos; a necessidade de usar diferentes meios de sustentação, em particular as oportunidades das tecnologias da informação e comunicação.

**Palavras-chave**
aprendizagem integrada de conteúdos e línguas; Aprendizagem Integrada de Matemática e Línguas; apoio educacional; materiais de aprendizagem; futuros professores de Matemática.

**Diseño instruccional de Aprendizaje Integrado de Matemáticas y Lenguaje (MLIL)**

para futuros profesores de Matemáticas

**Resumen**
Este artículo se centra en la problemática del Aprendizaje Integrado de Matemáticas e Inglés por parte de los futuros profesores de Matemáticas. Esta revisión de la literatura sobre el problema muestra una variedad de aspectos que los científicos consideran al estudiar el modelo mencionado. Sin embargo, investigaciones previas no han considerado en detalle el problema de los materiales didácticos apropiados para el Aprendizaje Integrado de Matemáticas e Inglés. El artículo muestra los resultados de dos encuestas realizadas con estudiantes de Matemáticas y Educación Secundaria (Matemáticas) en Ucrania (82 y 32 participantes, respectivamente), que ayudaron a extraer una conclusión sobre: la importancia del aprendizaje integrado para las matemáticas y los idiomas de los estudiantes; la importancia de los elementos del programa que reflejen los componentes matemáticos y lingüísticos de las metas, objetivos y resultados de aprendizaje esperados; la necesidad de que el contenido de Aprendizaje Integrado de Matemáticas e Inglés sea consistente con las necesidades actuales de los estudiantes; la necesidad de utilizar diferentes medios de apoyo, en particular las oportunidades de las tecnologías de la información y la comunicación.

**Palabras clave**
aprendizaje integrado de contenidos y lenguas extranjeras; Aprendizaje Integrado de Matemáticas y Lenguas; apoyo educativo; aprendiendo materiales; futuros profesores de Matemáticas.

1 **Introduction**

Integration and globalization processes of the socio-economic sphere of Europe and the world have a direct impact on changes in education. In recent decades, educators have reviewed approaches to the educational process, updating methods and techniques of teaching and learning, and looking for ways to improve the efficiency of the education. The modern educational environment is characterized by innovative methods of teaching and learning, aimed at cooperation between teacher and student, the development of skills of the 21st century, the involvement of the latest information and
communication technologies. The educators launch and implement the research projects using innovative approaches in order to promote the development of students’ competence based on participation and inclusiveness (MARTÍN; PÉREZ; ESTEBAN, 2017). Studying the application of innovations in education, scientists also investigate the role of a teacher in this process, teachers’ attitude to innovations in teaching and learning, and the results of using innovative methods in education (HASANOVA, 2022). One of the areas of innovation is Content and Language Integrated Learning (CLIL). It is defined by Coyle (2007) as a dual-oriented educational approach, in which additional language is used for learning and teaching content and language. This approach is relevant both in general secondary education in Ukraine in order to form foreign language communicative competence of students, and in training teachers, because they should implement this objective in teaching their subjects (Mathematics, Physics, Geography, Chemistry, etc.). We focus on some aspects of training Mathematics teachers in Ukraine using the CLIL methodology.

1.1 Background of the study. Content and Language Integrated Learning (CLIL)

CLIL aims (MALJERS; MARSH; WOLFF, 2007; MEHISTO; MARSH; FRIGOLS, 2008) at improving the study of both the language and the discipline studied by a foreign language. In this way, favourable conditions are created for the formation of both subject and language competence of students. Researchers note that the CLIL approaches are used in 30 European countries (CANADO, 2011). At the same time, the CLIL approaches differ significantly (COYLE, 2007; WOLF, 2002) in different countries due to the educational and linguistic characteristics of each country. Researchers distinguish (CANADO, 2011) the following common features of different implementations of the CLIL approach in higher education: the use of English as a foreign language; combination of a foreign language with a regional language; the scope of CLIL covering history, geography, natural and social sciences; the lack of the evaluation of the CLIL application. Research on teachers’ attitude to the implementation of the CLIL methodology (RISKHULOVA, 2021) shows that a significant number of teachers see greater prospects for the application of this approach in higher education, rather than secondary schools, and believe that the application of the CLIL approach...
has a positive effect on student motivation and learning outcomes in both subject and foreign language. Characterizing the CLIL methodology, we note the common features of CLIL with other approaches implemented in the modern educational paradigm. One of the characteristic features of CLIL is student-centered learning, which is also typical for task-based language learning (FROST; BRITISH COUNCIL, 2021; WILLIS, 1996). Elements of CLIL can also be observed in cooperative learning. Project-based learning technologies also comply with the basic principles of CLIL. The theoretical sources of inquiry-based learning are also similar to the CLIL methodology (YANG, W.; YANG, L., 2021). The common features of the above innovative approaches and CLIL prove the integrated interdisciplinary nature of the goals of the modern education nowadays, its focus on the development of student professional and social skills necessary for successful career development in the 21st century.

Research describing the features, processes and results of the CLIL implementation (CAPONE; SORBO; FIORI, 2017; FERNANDEZ; HALBACH, 2009; PARSONS; CALDWELL, 2016) shows its positive influence. Scientific findings of Japanese scientists (PARSONS; CALDWELL, 2016) argue that the combination of CLIL with problem-based learning has a positive effect on students’ motivation to learn English at university. A study by Italian scientists (CAPONE; SORBO; FIORI, 2017) shows that the combination of CLIL with flipped learning and project-based learning (FERNANDEZ; HALBACH, 2009) has a positive effect on the quality of the acquired subject knowledge of students.

1.2 Mathematics and Language Integrated Learning (MLIL)

The application of the CLIL approach in the study of Mathematics and a foreign language (MLIL) has been studied (ABEDI, 2001; BARWELL, 2009; CLARKSON, 1992; CLARKSON; DAWE, 1994; DALE; CUEVAS, 1987; ELLERTON; CLARKSON, 1996; NEVILLE-BARTON; BARTON, 2005) in different countries in the context of different problems: general pedagogical laws of construction of the educational process aimed at mastering the mathematical content by means of English, features of planning and designing teaching/learning, creating curricula and programs for teaching Mathematics by means of English, policy of assessing learning outcomes and achievements of students.
The positive effect of the approach on improving the level of mathematical competence of students in a relatively short period of time was found by Belgian researchers (SURMONT et al., 2016). Czech researchers (NOVOTNA; HOFMANNOVA, 2021) focused on such aspects of integrated teaching of mathematical content and English as: the interaction of three “languages” in teaching Mathematics (native language, foreign language and language of Mathematics), as well as the advantages and disadvantages of using CLIL in Mathematics.

We will start our work based on the conclusions reached by Czech researchers, namely, that learners always join the perceived stimuli with the knowledge and skills they have already stored in their mind. The perception process largely depends on the context. Learning isolated vocabulary items is unacceptable. The cognitive strategies are universal, not domain dependent. An analysis of the learning process of learning Mathematics and complementary language (English) by researchers from the UK (BARWELL, 2005) shows that a clearer reflexive model of the relationship between content, language and learning is needed.

Some researchers (HINESTROZA, 2019, p. 187) point to shortcomings in the implementation of MLIL and argue that:

Research on mathematics and language education in bilingual classrooms rarely maintains the interdisciplinary and interconnected nature of this phenomenon. Instead, research has tended to focus on language learning as a prerequisite for mathematics learning. Moreover, each of these two bodies of work is disseminated in its corresponding field, with few opportunities for interdisciplinary analysis.

Our observations also reveal some difficulties and reservations in the integrated teaching/learning Mathematics and foreign language. They are primarily related to the fact that the laws of cognitive processes that lead to the acquisition and successful use of language and mathematical content, being to some extent common, however, have their own specifics related to the subject area, the degree of abstractness of learning objects (mathematical and linguistic constructs). However, compliance with the laws of mastering the elements of mathematical content (formation of mathematical concepts, work with theorems and methods of mathematical activity) and elements of linguistic content (foreign language vocabulary, grammar, etc.) should lead to better results along with their coordination in learning and methodological support. To confirm this opinion, we rely on
research (AKBAROV; GONEN; AYDOGAN, 2018) at the Kazakh National University, which studied the experience of students in teaching Mathematics using the CLIL approach. These studies (AKBAROV; GONEN; AYDOGAN, 2018) analyzed the experience, results and opinions of students, their attitudes, preferences and perceptions of the educational process aimed at content and language integrated learning. The results of the study generally indicate an average level of satisfaction with this approach in the educational process, but there is an increase in the level of foreign language competence of students. At the same time, as the researchers (AKBAROV; GONEN; AYDOGAN, 2018) note, a positive and significant correlation of the level of English competence with the perception of Mathematics and other disciplines taught at the university in English was found.

Discussing various aspects of integrated teaching/learning language and Mathematics within the CLIL approach, the researchers point out the advantages of using a foreign language as the language of teaching Mathematics, as well as the advantages of using Mathematics as a means of teaching a foreign language. Possibilities of focusing on the linguistic and mathematical component provide different variations of CLIL-lessons (TEJKALOVA, 2013). Pointing to the advantages of the integrated learning of Mathematics and foreign languages, scientists (MIQDADI; DINA AL-LAMAL, 2013) emphasize that the main purpose of teaching mathematics is the formation of such qualities of personal thinking as consistency of argumentation, provability, alternative opinions (complete induction), system, structure, consistency in the process of expressing ideas and solving problems. These characteristics of mathematical thinking, according to experts, are enhanced if teaching Mathematics is combined with language learning. We share this vision of scientists. For example, the proof of a theorem (verbal or written) involves the construction of not only mathematical but also linguistic constructions for the logical ordering of a chain of purely mathematical arguments and conclusions. The logical action of defining mathematical concepts teaches students to such a way of expressing an opinion, when it is necessary to indicate only the essential properties of a certain class of objects, distinguishing them from insignificant ones. Students’ thinking in this case is aimed at performing such mental operations as comparison (in the form of comparison and opposition), analogy, classification, generalization, etc. The results of these operations are crystallized in the form of a
concept definition, which uses only the necessary and appropriate verbal constructions (necessarily logically consistent, devoid of descriptiveness, inaccuracy, metaphor).

Studying the role of a foreign language in teaching Mathematics, Italian scholars (FAVILLI; MAFFEI; PERONI, 2013) proceeded from a textual linguistic approach. Distinguishing four main types of mathematical discourse, namely: dialogic, descriptive, argumentative, and regulative-directive discourse (BENVENISTE, 1966; MACWHINNEY, 1995; SEARLE, 1969; WEINRICH, 2001) the researchers emphasize that the first three types can be used to analyze mathematical discourse in the classroom and, at the same time, represent the main linguistic aspects of communicative competence, namely: dialogue, description, narration. For example, the reasoned language in mathematics belongs to the category of narrative, but has its own mathematical specificity. The fourth type of discourse, regulatory and directive, is associated with a certain algorithmization in the formation and application of a certain method of mathematical activity (FAVILLI; MAFFEI; PERONI, 2013). In our opinion, these aspects must be taken into account when designing learning materials for MLIL. New Zealand researcher Latu (2006) studied the processes of students' comprehension of mathematical text, the influence of students' language difficulties related to the English language acquisition on the level of understanding and mastering mathematical content, the influence of language competence on students' mathematical competence in native language and foreign language. The main conclusions reached by New Zealand scientists as a result of their research are that students who use their native language alongside English, applying the phenomenon of “switching” from language to language, show better learning outcomes in learning Mathematics in English. The researchers emphasize the importance of teachers’ understanding that switching from language to language is a common practice of bilingual students. The results of this study formed the basis for our scientific research and practical development of educational support for integrated learning of Mathematics and English. Iranian researcher Clarkson (2008) focuses also on the process of “switching” students from one language to another while solving mathematical problems. The experts believe that the reasons for the difficulties in switching, for example, are due to the fact that earlier, students used the terms in their native language, and now they need to spend some time and correlate these terms with English equivalents. The researchers (CLARKSON, 2008) suggest that during the lesson, bilingual students will continue to use
the translation of certain concepts while performing mathematical tasks in order to understand them and better comprehend. Obviously, these problems are the basis for the use of native language in the CLIL model, as a kind of scaffolding for students at the stage of introducing new material or while proving the theorem.

The implementation of the MLIL model requires the provision of specific prerequisites and careful preparation. The research (TARASENKOVA et al., 2020) shows that MLIL in the professional training of the prospective Mathematics teachers should be implemented in two stages: successive models of learning profession-oriented foreign language (ESP) and, in fact, CLIL. The preparatory stages include the following steps: 1) determining the course for the application of this model; 2) selecting modules of the course; 3) planning the expected results; 4) selecting organizational forms and methods; 5) developing educational support for MLIL.

We have argued that an appropriate choice is the content of “Theory of divisibility in the ring of integers” and “Theory of congruence in the ring of integers” modules. It is because of several reasons. First, students showed a keen interest in learning these modules because of their widespread use in cryptology. An additional argument is that some concepts and theorems (the remainder theorem, the Fermat’s little theorem – a consequence of the Euler’s theorem –, the prime number theorem, the divisibility theorems) are represented in the advanced course of Mathematics in Ukrainian secondary schools. Many Ukrainian schools also offer students both advanced learning of Mathematics and English. Thus, the integrated teaching/learning of these modules corresponds to the current domestic educational trends not only in higher mathematical education, but also in secondary education. So, it is useful for future Mathematics teachers, along with Bachelors of Mathematics, to gain an experience in integrated teaching/learning of these topics. However, the question remains what should be the educational support for students and educators for successful integrated teaching/learning Mathematics and English.

Thus, the literature review on the problem of the article shows a variety of aspects that scientists consider when studying the integrated learning of Mathematics and English. However, previous research on MLIL did not consider the problem of selection and design of appropriate educational support in detail, namely learning materials (LM) for MLIL that would reconcile the patterns of acquisition of mathematical
knowledge and skills and the formation of foreign language competence of students – future mathematics teachers.

1.3 The purpose of the article

The purpose of the article is to identify the features of LM for MLIL on the example of teaching/learning „Mathematics in English (elements of divisibility theory and congruence theory)” for prospective mathematics teachers.

2 Methodology

To achieve the goal, we conducted two surveys of university students majoring in 111 Mathematics and 014 Secondary Education (Mathematics) in Ukraine (82 and 32 participants in the survey, respectively). They were carried out by the group of researchers from the Bohdan Khmelnytsky National University of Cherkasy. Standard closed ended questionnaire was developed. The data were collected within one month. Every participant filled in the questionnaire himself/herself. Each questionnaire took from 10 to 15 minutes to be filled out. The purpose of the study was explained to every participant, and we ensured participants that the information was confidential and would be used only for the purpose of the study.

3 Results and discussion

Our survey shows that 63.4% of respondents are interested in the integrated learning of Mathematics and English (Fig. 1). We conclude that the integrated learning of Mathematics and a foreign language is useful from the standpoint of both mathematical and foreign language training of future Mathematics teachers. At the same time, respondents prefer teaching/learning using MLIL technology while learning certain topics of a particular module (70%) or certain content modules of an educational mathematical discipline (61%). Therefore, we conclude that the content of MLIL (discipline “Mathematics in a foreign language”) should be varied, so that it is consistent with the current needs of students. It is important to note that almost 80% of respondents believe...
that the integrated learning of a particular Mathematics course and English requires partial (59.4%) or complete (21.9%) support in their native language. 100% of surveyed students consider it necessary to partially or completely duplicate the mathematical content in their native language on the lectures. At the same time, 45.2% of the respondents believe that the mathematical content should be partially duplicated in Ukrainian at the lecture; and 54.8% expressed a wish that the entire mathematical content should be duplicated.

Planning and development of LM for the integrated learning of the university discipline “Mathematics in English (elements of divisibility theory and congruence theory)” should: 1) ensure the implementation of different approaches to the introduction and consideration of previously known and new (to students) mathematical concepts, facts and methods; 2) take into account and combine the patterns of perception and assimilation of mathematical content (concepts, facts and methods of mathematical activities) and patterns of formation of foreign language competence through listening, reading, speaking, and writing; 3) implement new trends in teaching/learning caused by modern global processes and rapid changes in modern society, such as the spread of distance, blended learning, development of new educational services, mathematical software, etc. Elements of LM focus on the optimal planning and design of all stages of teaching/learning and the relevant types of educational activities. One of the main structures of LM for MLIL is a syllabus.

The syllabus reflects both the mathematical and linguistic components of the goals and objectives, and the expected learning outcomes. It is the characteristic feature of the MLIL syllabus in comparison with the syllabi of other disciplines. The syllabus states that the course is aimed at the formation of general and special competencies of specialists in relevant specialties, namely, the formation of the ability to operate with concepts, mathematical facts and methods of mathematical activities, to make a proof in different ways, to establish causal relationships, to demonstrate mastery of basic and special mathematical methods while analyzing and studying the problems of number theory in Ukrainian, and English. The linguistic component of the course is aimed at students’ mastering oral and written communication skills in Ukrainian and English to solve mathematical problems in number theory and the implementation of interpersonal and intercultural interaction. The course consists of basic mathematical concepts, facts
and methods related to the theory of divisibility and the theory of congruencies in the ring of integers, and implemented in the following activities: listening, reading, reproducing and applying orally and in writing in Ukrainian and English.

The program results, reflected in the syllabus, take into account the conclusions reached by Czech scientists Novotna, Hofmannova (2021). They point out that students must, on the one hand, freely operate in their native language with previously studied mathematics concepts and facts (in our case they are from the theory of divisibility in the ring of integers and the theory of congruencies). On the other hand, they need to learn English for professional purposes in advance.

The format of the university discipline “Mathematics in English (Elements of Number Theory and Congruence Theory)” includes lectures (14 hours), practical classes (16 hours), students’ independent work (30 hours), additional individual tasks (20 hours), consultations (10 hours), modular tests. At each lesson, students implement both mathematical activities (activities with previously studied concepts and facts and new ones on the theory of divisibility in the ring of integers and the theory of congruencies) and linguistic activities (on mastering the linguistic component of the content). The preparation work and course of lectures and practical classes is determined by what type of student activity (mathematical or linguistic) is leading in a particular lesson and what is accompanying.

Although the priority of a certain type of student activity in the classroom is to some extent conditional, we will consider the priority (leading) type of activity, as a result of which students’ mastering new mathematical concepts, facts, methods or linguistic knowledge and skills. If the subjects of study are familiar to students, and the activity is aimed at their consolidation and application, then this type of activity will be treated as an accompanying one.

Lectures in MLIL play an essential role due to their mathematical and linguistic content, the ability to focus on general mathematical approaches in proving theorems and constructing methods of mathematical activity for solving problems (method of mathematical induction, method of complete induction, constructive proof, proof based on contradiction, etc.). However, in our opinion, lectures in MLIL should have a familiar mathematical content, so the leading type of students’ educational and cognitive activities during lectures is an activity related to mastering the linguistic component of the course.
The accompanying type of students’ activities is mathematical activity which is connected with the repetition and application of the mathematical component of the LM. In this context, we consider it appropriate to refer to the model of flipped learning (AKÇAYIR, G.; AKÇAYIR, M., 2018; BHAGAT; CHANG, C. N.; CHANG, C. Y., 2016). We base on the concept of flipped learning as a learning style when a teacher provides to students learning materials in a video format or other supportive form to learn or review as their homework, a student then uses them in the next class time for in-class activities and problem-solving exercises (VELYCHKO et al., 2020). In designing and conducting lectures we also rely on the results of research by Italian scientists (CAPONE; SORBO; FIORI, 2017) who substantiated the combination of CLIL with flipped learning, as well as scientists who prove measurable improvements in student and teacher motivation, increased attendance in class, and better grades, as a result of using the flipped approach (BISHOP; VERLEGER, 2013; DAVIES; DEAN; BALL, 2013; HAMDAN et al., 2013). We offer mathematical content for students’ independent review, and mastering the linguistic component occurs during lectures, because we provide activity of students through active listening, speaking (listening, repetition), reading, discussion at the lecture. When designing a lecture, we also take into account the wishes of students identified during the survey.

Learning the linguistic component of the educational content during the lecture is based on the previous students’ knowledge of general English and English for professional purposes. It is provided by the perception of oral (listening) and written speech (reading) during the lecture. Essential features of listening and reading are as follows: 1) these are receptive types of speech activity that are aimed at the perception of information implying the simultaneous perception and understanding of what is heard, read, or written; 2) the form of course is internal, which is based on the formed ability to understand what is heard read, or written; 3) the subject is someone else’s opinion, which is encoded in the written text or audiotext, which should be recognized; 4) the result is an inference; 5) the result is listening comprehension or reading comprehension and one’s own lingual and non-lingual behaviour.

We can forecast the difficulties for students and choose ways of scaffolding, i.e., ways to support, help and facilitate the perception of mathematical content presented in English, considering the properties of these types of speech activities. We take into
account that success of the perception of mathematical material in English depends primarily on the students, namely, on their listening comprehension skills, memory, attention, interest, that is, on the students’ individual psychological characteristics. Second, the success of perception and understanding depends on the mathematical text (information) and the way it is presented if it is new to students. If the mathematical content is familiar to students, then its perception depends on the level of their pre-formed mathematical ideas, knowledge and skills.

Therefore, we consider that variations of scaffolding at this stage of formation and development of listening and reading skills are obligatory. They will include: 1) prior knowledge (to start the learning process with previous students’ linguistic knowledge and use them as a basis for perception and understanding of new learning material); 2) pre-teach vocabulary (introduction of new lexical units in a mathematical context); 3) visual aids (use of formulas, graphs, diagrams that facilitate the perception and understanding of new or familiar mathematical content); 4) pauses, questions, reviews (accompanying the introduction of new mathematical content with pauses for comprehension, questions to control comprehension, repetition to control perception).

We consider how these techniques have been implemented in the development of educational support for the lecture “Divisibility in the ring of integers”. Students previously repeat the basic concepts and properties of divisibility of integers known to them. During the lecture, it is advisable to repeat them in the form of an interactive cloud of words. The formulation of the division theorem with remainder deserves more particular attention. At the same time, new lexical units of mathematical content are introduced. The introduction of new lexical units involves the perception of oral (listening) and written speech (reading). As a result of the introduction of new lexical units, students consider and comment on a new interactive cloud of words, composed of well-known mathematical terms given in English. In designing LM for lectures, it is necessary to consider the possibilities of the latest educational technologies, the opportunities of educational information and communication technologies (ICT) to support the educational process, that have a significant impact on ensuring MLIL. They allow us to diversify ways of participants’ (students, teachers and the modern educational environment) interaction. The model of blended learning allows us to realize these possibilities of new modern ICT. This model of teaching and learning allows us (VELYCHKO et al., 2020) to take an
advantage of face-to-face learning and electronic educational resources through a combination of distance and traditional communication in integrated learning activities. The integration of traditional and computer-based learning will lead to a purposeful acquiring knowledge, skills and abilities in the classroom and extracurricular learning activities through the use of ICT. The existence of this form of learning is possible due to the effective combination of different ways of presenting educational content, teaching models and styles. It is based on the interaction between all participants of the educational process. These aspects must be taken into account in the design and development of LM for lectures of the course “Mathematics in a foreign language (elements of number theory and congruence theory)” in MLIL. Therefore, we offer to include interactive exercises created with the help of LearningApps. The next stage in the formation and development of linguistic skills is to work with the text, in our case, with the proof of the theorem on division with remainder. We offer for consideration a variant of proof that differs from the previously studied in some aspects, although the general method of proof (constructive proof) remains similar to that previously studied by students. Organizational forms of this work include work in pairs (read and discuss), in mini-groups and in a group (read and talk about the main idea of the certain step of the proof). The exercise is used to control reading comprehension. At the same time, the implementation of task 4 provides some assistance to students using the scaffolding technique. It involves the application of the prior knowledge of students to prove the theorem in their native language, using an interactive whiteboard Jamboard (Fig. 1).

**Figure 1** – Using an interactive whiteboard to review previously learned material
The final stage of forming the ability to prove a theorem in English is task 5. It involves writing a proof of the theorem in English, using proof in Ukrainian as a support. Lexical material in English is used too to formulate students’ thoughts and conclusions.

To ensure the accompanying mathematical activities, we formulate tasks (Fig. 2) in English in order to apply the obtained mathematical and linguistic knowledge in practice. Tasks are performed by the students in the collective activity under the teacher’s control and with the teacher’s help. The final stage of the lecture, in our opinion, involves the transfer of acquired knowledge and skills of students into new conditions. For this purpose, we offer creative work to students in pairs or mini-groups. It covers the development of skills of clustering concepts and mathematical facts by means of English, using the Brainstorm approach. In this way, the formation of flexibility as a necessary quality of linguistic skill is ensured.

Figure 2 – Math and English in Practice

4 Final considerations

Thus, the integrated learning of mathematics and English contributes to both mathematical and language training of future Mathematics teachers. The content of MLIL should be consistent with the current needs of university students. LMs in MLIL are intended to support students and teachers, to develop students’ content knowledge and language skills. LM should include syllabus which reflects both the mathematical and linguistic components of the goals and objectives, and the expected learning outcomes. LMs in general promote students to comprehend, conceptualise, systematise, appreciate
and contemplate mathematical concepts and theorems as well as the experiences in speaking and writing in English. LMs for lectures in MLIL are grounded on a familiar mathematical content, so the leading type of students’ educational and cognitive activities during the lectures is an activity related to mastering the linguistic component of the course. Learning the linguistic component of the educational content during the lecture is based on the previous students’ content knowledge and knowledge of general English and English for professional purposes. Different means of scaffolding are obligatory. They will include: 1) prior knowledge; 2) pre-teach vocabulary; 3) visual aids; 4) pauses, questions, reviews. We ensure that it is necessary to partially duplicate the mathematical content in the native language at the lectures. In designing LM for lectures in the MLIL model, it is necessary to use the opportunities of ICT that have a significant impact on ensuring MLIL. As far as the authors know, this article is one of the few articles (MEHISTO, 2012; NEVILLE-BARTON; BARTON, 2005) that focuses on the problem of MLIL in this context.

5 References


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