

Multiplicative structures in the form(action) of teachers from elementary school in a school of Fortaleza

Antonio Luiz de Oliveira Barretoⁱ 

State University of Ceará, Fortaleza, CE, Brazil

Rogéria Gaudencio do Rêgoⁱⁱ 

Federal University of Paraíba, João Pessoa, PB, Brazil

Abstract

In order to better understand the knowledge of teachers who work in elementary school about the Multiplicative field, this study aims to map and analyze the types of multiplication problems produced by them. Fifteen teachers participated in the study, and each of them created six problems involving multiplication and/or division. In the analysis, we considered as reference the axes proposed by Magina, Merlini and Santos (2016). Out of the 90 presented questions, 75 were classified as multiplication problems and restricted to three axes: simple proportions, multiplicative comparison, and product of measures. No problems involving double proportion, multiple proportion or rectangular configuration were suggested, indicating a limited repertoire of multiplicative situations by the teachers. The results point to a need for investment in continued training that explores the study of multiplicative problems in the axes that were not considered by the teachers and the expansion of those present in the proposed problems.

Keywords

Teacher training. Conceptual Fields Theory. Multiplicative Structures.

Estruturas Multiplicativas na form(ação) de professores dos anos iniciais do Ensino Fundamental de uma escola de Fortaleza

Resumo

Visando entender melhor o conhecimento de docentes que atuam nos anos iniciais do Ensino Fundamental sobre o Campo Multiplicativo, este trabalho objetivou mapear e analisar os tipos de problemas multiplicativos elaborados por eles. Participaram do estudo 15 professores, tendo cada um deles elaborado seis problemas envolvendo multiplicação e/ou divisão. Na análise das situações-problema, foram considerados como referência os eixos propostos por Magina, Merline e Santos (2016). Das 90 questões apresentadas, 75 foram classificadas como problemas multiplicativos e restritos a três eixos: proporções simples, comparação multiplicativa e produto de medidas. Nenhum problema de proporção dupla, de proporção múltipla nem da classe configuração retangular foi sugerido, desvelando um restrito repertório de situações multiplicativas dos docentes. Os resultados apontam para a necessidade de investimento em uma

formação continuada que explore o estudo de problemas multiplicativos dos eixos que não foram considerados pelos professores e o aprofundamento dos eixos presentes nos problemas propostos.

Palavras-chave

Formação de professores. Teoria dos Campos Conceituais. Estruturas Multiplicativas.

**Estructuras Multiplicativas en forma(acción) de docentes de los años
iniciales de la Enseñanza Fundamental de una escuela de Fortaleza**

Resumen

Con el fin de entender mejor el conocimiento de los docentes que actúan en los años iniciales de la Educación Primaria sobre el Campo Multiplicativo, este trabajo pretendió mapear y analizar los tipos de problemas multiplicativos elaborados por ellos. Participaron del estudio 15 docentes. Cada uno de ellos elaboró seis problemas relativos a la multiplicación y/o a la división. En el análisis, se consideraron como referencia los ejes propuestos por Magina, Merlini y Santos (2016). De las 90 preguntas presentadas, 75 fueron clasificadas como problemas multiplicativos y restringidas a tres ejes: proporciones simples, comparación multiplicativa y producto de medidas. Ningún problema de proporción dupla, de proporción múltipla ni de clase de configuración rectangular fue sugerido, revelando un repertorio restringido de situaciones multiplicativas de los docentes. Los resultados apuntan a la necesidad de inversión en una formación continua que explore el estudio de problemas multiplicativos de los ejes que no fueron considerados por los profesores y la profundización en los ejes presentes en los problemas propuestos.

Palabras clave

Formación de docentes. Teoría de los Campos Conceptuales. Estructuras Multiplicativas.

1 Introduction

According to Pinheiro *et al.* (2018), the main professional action by pedagogue teachers in Basic Education, especially those who work in Early Childhood Education and Elementary School, happens in school, which, as an institution, is the main locus to acquire knowledge (BEGO, 2016; GENÚ, 2018). According to Maia *et al.* (2015), those educators show gaps in training regarding Math concepts. Many graduated from Pedagogy courses, which, generally, offer few disciplines aimed at teaching Math. “The result from this scenario is the continuance of limited teaching concepts and practices,

which contribute little to the development of abilities and competencies in Math that are necessary to learners” (MAIA *et al.*, 2015, p. 2225, our translation).

In the classroom, the concept of multiplication is introduced through the idea of repeated addition of equal values (NUNES *et al.*, 2009), and the teacher, in this perspective, working on problems with the students, promotes connections between addition and multiplication. However, from a conceptual point of view, there is significant difference between additive reasoning and multiplicative reasoning.

According to Nunes *et al.* (2009), additive reasoning is based on a single conceptual invariant, the part-whole relation. The parts are known and the whole is sought. Or, in turn, the whole and one of the parts are known and the other part is sought. “In contrast, the conceptual invariant of multiplicative reasoning is the existence of a fixed relation between two variables (or two measurements or quantities). Any multiplicative situation involves two quantities in constant relation with each other” (NUNES *et al.*, 2009, p. 85, our translation). Besides, the concepts of addition and subtraction are originated from the action schemes of joining, separating and one-to-one correspondence, while the concepts of multiplication and division are originated from the schemes of one-to-many correspondence and distribution.

The connections and ruptures between additive and multiplicative reasoning have been researched in depth by Vergnaud (1983, 2009) in the Conceptual Fields Theory. The Multiplicative Structures involve a group of situations whose mastery requires an operation of multiplication or division or a combination of those and many concepts: multiple, divisor, fraction, simple and multiple proportions, linear, compound and bilinear functions, among others.

This study focuses on the Multiplicative Field and aims to map and analyze Math problems created by Elementary School teachers before continued training. It is part of an ongoing research for the post-doctorate program attended by the first author of this text, under guidance of the author who accompanies him in this article, which investigates the contributions of a continued training, in a collaborative perspective, to the pedagogical practice of Elementary School teachers, regarding the development of the covariation concept present in multiplicative structures in the context of the axis of simple proportions.

We emphasize that collaborative training doesn't privilege the vertical relationship between participants, but instead constitutes "[...] a partnership between educators and students, who can interact collaboratively and are co-responsible for problem-solving, challenges of the practice and joint production of knowledge concerning educational practice" (SANTOS, 2012, p. 30, our translation).

In accordance with that idea, we bear in mind that a continued training course must be a space for constant reflection about the practice in the classroom, fostering reflexive learning habits and idea sharing, valuing an effective experience exchange. It is necessary to have a training space where future teachers can "[...] be heard about their own practice, which can provide transformation, development or even improvement in their practice" (JUNGES; KETZER; OLIVEIRA, 2018, p. 97, our translation).

Through the analysis of problems within Multiplicative Structures, Vergnaud (2009) classifies situations according to characteristics and complexity, grouping them in: isomorphism of ideas, case of a single measuring space and product of measures. In our research, we used the classification of Multiplicative Structure situations proposed by Magina, Merlini and Santos (2016), who detailed the organization, division and classification of Math problems from the Multiplicative Conceptual Field, as will be presented.

According to Magina, Merlini and Santos (2016), multiplicative problems are divided into quaternary relations and ternary relations. Quaternary relations consist of three axes: simple proportion, double proportion and multiple proportion. Each axis is subdivided into classes (one to many and many to many). Ternary relations are organized in two axes: multiplicative comparison and product of measures. The multiplicative comparison axis consists of the classes referent unknown and relation unknown. The product of measures axis has the classes rectangular configuration and combinatory configuration. Except for the combinatory class, each class is divided into two types of quantities: discrete quantity and continuous quantity. The combinatory class only involves discrete quantities. We also adopt the classification by Gitirana *et al.* (2014) for the Multiplicative Structure problems, according to the degree of cognitive complexity.

Gitirana *et al.* (2014) present a classification for Multiplicative Field problems according to their level of difficulty, which can be prototypes or extension. The former are

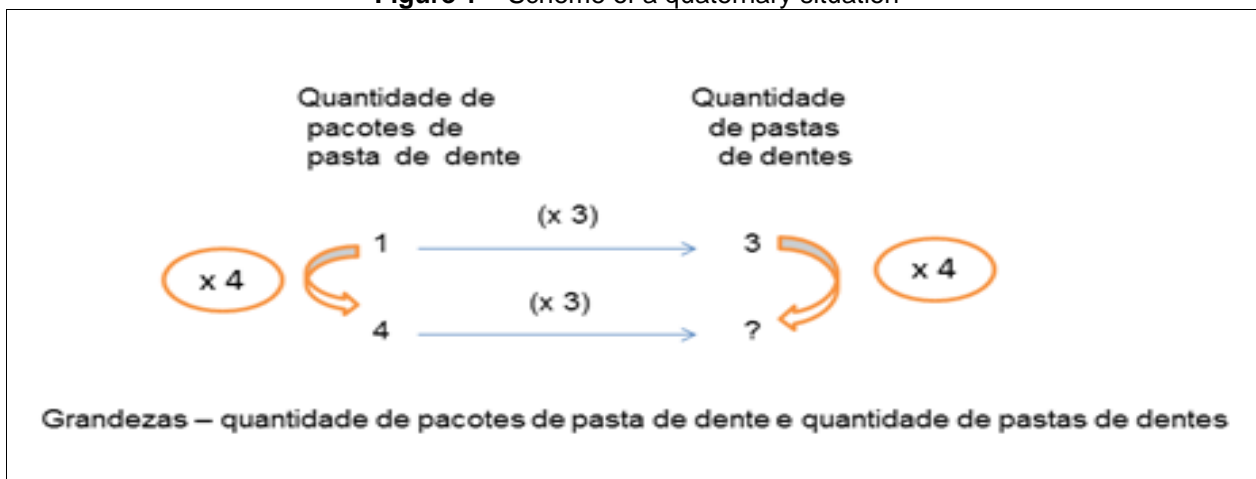
those that require simpler reasoning and children have no difficulty to solve them, while extension problems demand more elaborate forms of thought from the students. The extension problems grow increasingly difficult in an extensive scale from 1st to 4th extension.

Before we discuss each axis, we will define a succinct distinction between ternary and quaternary relations. Ternary relations are those that connect three elements among themselves (VERGNAUD, 2009), for example: “Isa’s house has a rectangular shape 11 meters long and four meters wide. What is the area of Isa’s house?”. To solve that problem, we must consider three measures: the width of the house (4 m); the length (11 m); and the unknown area.

The quaternary relation is defined as a connection between four elements among themselves and “[...] frequently has the following form: ‘a is to b as c is to d’. It reaffirms that the relation between a and b is the same as between c and d” (VERGNAUD, 2009, p. 71, our translation). We have the following problem as example: “Eli has four packs of toothpaste. There are three tubes of toothpaste in each pack. How many tubes of toothpaste does Eli have?”.

According to Santos (2012), this type of multiplicative problem is usually solved in school through a ternary situation: $a \times b = c$ ($4 \times 3 = 12$). However, in this situation, implicitly, we have a quaternary relation between two quantities of different natures that can be schematically represented as shown on Figure 1.

Figure 1 – Scheme of a quaternary situation



Source: Elaborated by the authors (2019).

In this case, we have a double relation between two quantities (packs and tubes of toothpaste). Understanding quaternary relations gives the student the reasoning behind the action of multiplying the number of packs for the number of tubes and the result being the number of tubes, and not packs (SANTOS, 2012). Also, through the quaternary relation, the teacher can discuss with the students the strategy of using the multiplicative scale factor (“ $\times 4$ ” in Figure 1) or the functional factor (“ $\times 3$ ” in Figure 1) to find the number of tubes of toothpaste.

We don’t intend to explore deeply or exhaust the discussion about the classification proposed by Magina, Merlini and Santos (2016) about Multiplicative Structures in this article. Except for the simple proportion axis, we won’t discuss the many to many class in quaternary relations or the types of quantities (discrete or continuous). For more details, see the article by those authors or by Santos (2012).

The simple proportion problems belong to quaternary relations and “[...] bring situations in which there is a proportionality relation between four measurements, two in two, of the same type – which are related through a rate between the measurements of different types” (GITIRANA *et al.*, 2014, p. 55, our translation). The following example illustrates the simple proportion axis: “One car has four tires. How many tires do five cars have?”.

We notice in that example that increasing the number of cars will proportionally increase the number of tires in a fixed relation of 1:4. Double proportion problems present situations involving, at least, three measurements of different natures. In the particular case of three measurements, we have two simple proportions consisting of three measurements, in which two are proportional to a third one, but not among themselves. The following problem illustrates this axis: “One person eats 250 grams of meat per day. How many grams of meat will a family of five people eat in ten days?”.

In this problem, the measurement meat consumption in grams is directly proportional to the number of days and the number of people. However, the latter two aren’t directly proportional to each other, insofar as, if we vary the number of people, the number of days won’t be changed.

The problems of multiple proportion involve more than two measurements, related two in two, in a quaternary relation, proportional among themselves. It is worth noting that, in this case, unlike double proportions, if we change the value of any measurement involved, all others will be modified (GITIRANA *et al.*, 2014), as in the example: “Ms. Teresinha’s cake recipe is this: for each cup of milk, she uses three cups of flour; for each cup of flour, she adds two cups of sugar. If she uses seven cups of milk, how many cups of sugar will she use?”.

Analyzing the example, we verify that the measurement number of cups of milk is directly proportional to the measurement number of cups of flour and number of cups of sugar, and the latter two are directly proportional to each other. We notice that, if we change any of them, all the others will necessarily change. As we emphasized, all axes of quaternary relations involve classes one to many and many to many.

Chart 1 contains the question from three simple proportion problems of the type one to many.

Chart 1 – Questions from three problems of the type one to many

Problem 1	Problem 2	Problem 3
Sá loves ice cream cones. We know that each ice cream cone costs R\$ 2.00. How much will Sá pay if she buys four ice cream cones?	Sá bought four ice cream cones for R\$ 8.00. How much did she pay for one ice cream cone?	Sá has R\$ 8.00. How many ice cream cones can she buy if each ice cream costs R\$ 2.00?

Source: Elaborated by the authors (2019).

Problem 1 is a simple proportion of multiplication one to many. In measurement isomorphism, it’s a simpler problem, as far as four amounts are put in play, but one of those amounts is equal to one (VERGNAUD, 2009). We have the cost of one unit (the price of one ice cream cone is R\$ 2.00) and we want to discover the cost of many units (four ice cream cones).

The simple proportion problems of multiplication one to many are prototypical, don’t cause difficulties for Elementary School students (GITIRANA *et al.*, 2014). Many students, based on additive reasoning, can solve them using the strategy of adding equal amounts. However, it is possible to vary that problem, informing the value corresponding to certain quantity and asking students to calculate the value corresponding to the unit (partition problems).

Problem 2 is a partition problem: to find out how much Sá paid for one ice cream cone, we divide the total paid (R\$ 8.00) by the number of ice cream cones (four cones), i.e., $8 / 4 = 2$. In the resolution of partition problems, the reasoning used is associated to a division of amounts of different natures and to division as an idea of sharing, distributing or splitting (GITIRANA *et al.*, 2014; SANTOS, 2012).

Another variation of problem in the class one to many is the quota problem (measurement problem). Problem 3 is a quota problem. In this problem, we have the value corresponding to the unit (one ice cream cone costs R\$ 2.00) and a given amount (Sá has R\$ 8.00 to buy ice cream cones) and want to find out how many ice cream cones can be bought with R\$ 8.00. In this sense, we carry out the division of two amounts of the same type: $8 / 2 = 4$. Writing in another way, we want to know how many quotas or groups of R\$ 2.00 can be obtained with R\$ 8.00. The answer consists of four groups.

Regarding the analysis of cognitive difficulties, quota problems are more complex than partition problems. Dividing two measurements of the same nature, the result will be a dimensionless number. In the case of the problem analyzed here, the student should notice that $R\$ 8 : R\$ 2 = R\$ 4$, corresponding to the number of the other measurement, number of ice cream cones. Quota and partition problems are classified, respectively, as first extension and prototype (GITIRANA *et al.*, 2014).

We saw that, in situations of the class one to many, a unit of one measurement is associated to many units of the other measurement. Whereas in situations many to many, the relation of proportionality is maintained, but the unit isn't the same as one of the elements involved in the situation. Many to many correspondence problems are also called fourth proportional. Let's see an example of problem from that class: "Eli went to the fabric store and bought 12 meters of linen for R\$ 156.00. How much would Eli pay for 25 meters of linen?"

We observed that this situation is more complex than problem 1 (relation one to many). In fact, in the example given, the two known values of the same measurement (quantity of linen) aren't multiples (12 is not a divisor of 25). Among possible strategies for the resolution, the student can use the one based on discovering the value corresponding to the unit, dividing 156 by 12 and obtaining 13. Then, they multiply 13 by 25, obtaining 325. "Thus, they use an intermediary stage.

First, they find the value of the unit, as if solving a partition problem. Second, with the value for the unit, they solve the problem as if it were a situation of one to many” (GITIRANA *et al.*, 2014, p. 67, our translation).

The problem-solving process for the fourth proportional, especially those in which the values for the same measurement are known and aren’t multiples of each other, is more complex than the process used in simple proportion problems of multiplication one to many, demanding higher cognitive effort from the student. According to Gitirana *et al.* (2014), fourth proportional problems are second extension.

Multiplicative comparison problems are ternary, involving two variables of the same nature (referent and referred), which are compared to each other through a multiplicative relation (scale). Examples of problems of multiplicative comparison situations are shown on Chart 2.

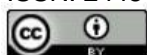
Chart 2 – Examples of three multiplicative comparison problems

Problem 4	Problem 5	Problem 6
I went to a store and bought a rock CD for R\$ 19.00 and a jazz DVD for R\$ 38.00. How many times was the jazz DVD more expensive than the rock CD?	Eli and Rui collect soccer cards. We know that Eli has 50 cards and Rui has twice as many cards as Eli. How many cards does Rui have?	I bought a ball and a toy car. We know that the car was four times more expensive than the ball. The toy car cost R\$ 80.00. How much did the ball cost?

Source: Elaborated by the authors (2019).

Problem 4 belongs to the unknown relation class. Analyzing it, we recognize the referent (price of the rock CD) and the referred (price of the jazz DVD), asking to find the relation (how many times more) that exists between those two values. To solve it, we need to do the following operation: $38 : 19 = 2$. Since the problem involves an inversion of operation, Elementary School students have difficulty to solve it. Gitirana *et al.* (2014) classify it as third extension with an elevated degree of complexity.

Problem 5 belongs to the class unknown referred: we know the referent (number of cards Eli has) and the relation (twice as many) and require the determination of the referred (number of cards Rui has). To solve it, we need to do the following operation: $50 \times 2 = 100$, i.e., it is a prototypical situation of multiplication.



Problem 6 involves calculating the unknown referent. To solve it, we turn to the following operation: $80 : 4 = 20$. Since it is an inverse situation, the resolution difficulty degree is greater than the previous problem (Problem 5), being classified as a second extension problem (GITIRANA *et al.*, 2014).

The axis product of measurements consists of two classes: (a) situations involving the idea of rectangular configuration and (b) situations involving the idea of combinatory. Rectangular configuration is a class of situations that “[...] involves the idea of rectangular organization, which enables their resolution through the mathematical model ($a \times b = c$ or $c : a = b$)” (SANTOS, 2012, p. 119, our translation). Chart 3 contains examples of rectangular configuration problems.

Chart 3 – Examples of rectangular configuration problems

Problem 7	Problem 8
We know that a plot of land located at a beach has a rectangular shape with 10 meters of width and 12 meters of length. Calculate its area.	The area of Maria’s farm is rectangular and measures 168 square meters. The width measures 12 meters. What is the farm’s length in meters?

Source: Elaborated by the authors (2019).

Solving problem 7, we turn to the following operation: $A = 10 \text{ m} \times 12 \text{ m} = 120 \text{ m}^2$. In problem 8, in order to calculate the farm’s length, the operation required is division ($c = 168 \text{ m}^2 : 12 \text{ m} = 14 \text{ m}$). The rectangular configuration axis can also involve problems concerning volumes (VERGNAUD, 2009).

Combinatory involves the concept of combination, and that class has the notion of Cartesian product between two finite and disjointed sets. It is possible to obtain the product of the number of elements in set A for the number of elements in set B, thus determining the number of elements of a new set. In Chart 4, we present two examples of combinatory problems, respectively called part-part combinatory problem and part-whole combinatory problem (SANTOS; MERLINE, 2018).

Chart 4 – Examples of combinatory problems

Problem 9	Problem 10
<p>At a diner, there are two types of sandwiches (meat and cheese) and three types of smoothies (banana, papaya and strawberry). In how many different ways can I obtain a different meal containing only one sandwich and one smoothie?</p>	<p>A diner serves 12 different meals. For each meal, they use only one sandwich and one juice. Knowing that the diner offers three different types of sandwiches (meat, cheese and chicken), how many different types of juice are necessary to assemble all the types of meals?</p>

Source: Elaborated by the authors (2019).

In problem 9, we have the parts, the number of sandwiches and smoothies, and seek the whole, which is the total number of meals formed with the two parts. To solve it, we effect a multiplication between the number of sandwiches and the number of smoothies. Thus, the number of different meals is 2 sandwiches \times 3 smoothies = 6 meals. We can also solve it through a diagram or double-entry table (SANTOS, 2012; SOUZA, 2015). To solve problem 10, we use the inverse reasoning and, for that, use a division.

2 Methodological decisions: procedures and tools

Our post-doctorate research was divided in two parts, with the first part accomplished before training and the second part carried out during a continued training course with Elementary School teachers who teach Math. In the first part, we applied data collection tools to ground the other research stages, as well as to adjust the initial training proposal. The second part consisted of the continued teacher training, carried out at an Elementary School in the city of Fortaleza, Ceará. In this study, we focus on the results from the first phase of research, i.e., the analysis and discussion of two data collection tools. The first tool consisted of a questionnaire aiming to understand the participants' professional profile (academic training, career duration, number of classes taught).

The second tool asked teachers to create six questions about Multiplicative Structures, aiming to understand the types of problems that they usually solve with their students. They received the following instruction, along with six numbered rectangles:

“Create, in the spaces below, six different problems involving multiplication and/or division”. The teachers couldn’t consult a Math textbook, their lesson plans or any sort of digital tool, and they had to create the problems individually. As they finished, they handed the problems to the researchers.

The study subjects were 15 teachers graduated in Pedagogy, History and Geography. All participants work in Elementary School. To ensure anonymity, teachers and their tools were identified as P1, P2, ..., P15.

For this analysis, we separated the questions proposed by the 15 teachers, categorized according to the five axes of Multiplicative Structure problems presented by Magina, Merlini and Santos (2016). Since all teachers created six questions, we had a total of 90 questions to analyze.

3 Results and discussion

Among the 90 problems created by the teachers, 15 were considered inadequate and 75 were adequate. Questions considered inadequate are divided in two groups. The first group involves non-multiplicative situations (33.33%). The second group of inadequate problems has situations without meaning or incomplete questions (66.67%). The ten problems without meaning or with incomplete questions were created by five of the 15 teachers, evidencing a limited repertoire of Multiplicative Structure problems and gaps in their Math education. In this context, we can infer that this gap can impact the classroom, when the teachers solve with the students problems whose questions have linguistic imprecision, impairing the students’ learning (MAIA *et al.*, 2015).

We present, in Chart 5, two problems considered inadequate:

Chart 5 – Examples of problems created by the teachers

Problem 11	Problem 12
Mariana has 12 stamps and got 48 more stamps from her mother. How many stamps does she have in total?	Raimundo went to the store to buy two thousand bricks, to use in a week, but he has to use them gradually. How many bricks will he use each day?

Source: Elaborated by the authors (2019).

Problem 11, created by teacher P15, who has 22 years of career and teaches 5th grade, is a non-multiplicative situation whose solution involves an addition. Analyzing the group of problems she created, out of six questions, three were classified as from the additive field.

Two other teachers also created additive problems, although we emphasized in the request that the problems to be proposed should involve multiplication and/or division. These data converge to the research by Maia *et al.* (2015). In the question of problem 12, there is a lack of relevant information, since we can't know how many bricks will be used each day nor what "gradually" means. Due to the lack of clarity in its creation, the resolution may have many answers.

Analyzing the adequate situations, we emphasize the predominant presence of simple proportion problems. Out of 75 problems created by the teachers that were considered appropriated to what was requested, 69 had that nature, corresponding to 92% of proposed problems. Among the other six problems, five were about multiplicative comparison, around 6.67% of the total, and one involved product of measurements, precisely 1.33% of the total. The other axes – double proportion and multiple proportion – weren't contemplated.

Based on these results, we can infer that teachers have very limited knowledge about the types of problems from the multiplicative conceptual field. This may have an impact in the classroom, insofar as the teachers will explore prototypical problems with the students, more simple and common, without the recommended diversification.

The 69 simple proportion problems were of the type one to many, therefore, no many to many problems were presented. These results are similar to those obtained by Maia *et al.* (2015). We agree with those authors when they state that "[...] this favors the continuance of the wrong perception that multiplicative problems of measurement isomorphism are ternary" (MAIA *et al.*, 2015, p. 2232, our translation).

Out of five problems proposed about multiplicative comparison, all belonged to the class "referred unknown", revealing a limited group of questions relative to that axis. To solve them, we should turn to a multiplication. As we saw, this class of problems is classified by Gitirana *et al.* (2014) as prototypical, which are those with an easy solution.

Among the cited questions, we identified two groups of problems. The first group explored expressions such as “double”, “triple”, thus in accordance with the National Common Curricular Base (BNCC, in Portuguese), which indicates the exploration, since the 2nd grade of Elementary School, of problems involving those meanings (BRASIL, 2018). The second group of problems involved expressions such as “times more”, without problems using the expression “times less”, revealing a gap in the creation of problems of that class.

Problems that explore the expression “times less” are more cognitively complex than those who use the expression “times more”. In fact, the learner, when faced with the expression “times more”, can associate it to a multiplication. However, the second expression, “times less”, is far from taking on the meaning of division. Sometimes, the student, after reading that linguistic expression, may effect, instead of division, a multiplication and, then, a subtraction (BARRETO *et al.*, 2017).

Having as a focus of analysis the product of measures axis, we observed a dearth of questions exploring rectangular configuration, involving problems relative to area and volume, subjects in which Elementary School teachers, generally, have difficulties (SILVANA, 2018).

There was only one problem regarding combinatory class, created by teacher P14, of the type part-part. Concurrently, we noticed a lack of part-whole combinatory problems, involving the opposite relation, which limits children’s education in the classroom.

Analyzing simple proportion problems, the data revealed that, out of all problems created by the teachers, 52.17% were partition and 43.48% were simple proportion problems of one to many multiplication, distantly followed by quota-type problems (4.35%). Regarding problems involving partition and quota, these results are in accordance with those obtained by Souza (2015). In that study, among questions about simple proportions that explored division created by teachers, the majority were partition problems, to the detriment of the low number of quota problems presented.

In our study, quota problems were considered of first extension, which are harder than partition problems. “It is likely that there is a relation between this complexity and the low number of problems proposed” (MAIA *et al.*, 2015, p. 2234, our translation). On the other hand, “[...] the emphasis given by the school to the

approach of division as partition ends up promoting a barrier for students to identify the meaning of division as quota” (GITIRANA *et al.*, 2014, p. 62, our translation). It is important to emphasize that, in the traditional division algorithm, the predominant model is the quota, in which we seek to identify how many times a number “fits” into another, which is, according to Van de Walle (2009), a “very mysterious process for children” and may hinder their understanding.

4 Final considerations

Through the analysis of problems created by Elementary School teachers with the request that they were from the Multiplicative Field, we aimed to identify teachers’ knowledge about problems from that field and identified some points that should be highlighted. In the simple proportion questions, there was a preponderant incidence of partition problems to the detriment of quota (measurement) situations, revealing a limited repertoire of problems of that nature, in turn revealing gaps in the teachers’ education.

In the classroom, we emphasize that, according to Vergnaud’s principles, teachers should work with both partition and quota problems, fostering children’s learning through the exploration of a greater diversity of situations. This approach would be in accordance with what the authors we adopted recommend and, more recently, with what is recommended by the BNCC’s proposal for Math, working, in the 3rd grade of Elementary School, with the different meanings of division and multiplication, among which the meanings of dividing in equal parts (equitable) and the meaning of measure (BRASIL, 2018).

In the multiplicative comparison axis, only unknown referred problems were created, whose resolution was based on a direct operation, i.e., a multiplication. It is necessary to expand children’s experience with the Multiplicative Field, working with situations that involve the inverse meaning, turning to the division operation.

Also, no teachers created problems of double proportion or multiple proportion. We can explain that fact with the following reason: problems in those two axes are more explored in the classroom after the 7th grade, which opens a possibility that they

aren't part of the repertoire of multiplicative problems of the teachers researched (GITIRANA *et al.*, 2014; SANTOS, 2012).

In the product of measures axis, only one teacher created a combinatory problem of a part-part type, without any questions in that axis using the inverse reasoning. The class of rectangular configuration problems wasn't contemplated. The results obtained in the initial part of our research were essential to restructure training plans, leading us to emphasize other multiplicative situations and promote further understanding about those that were created by the teachers who participated in this research.

However, “[...] it is not merely about instrumentalizing them with a larger repertoire of multiplicative structure problems, but helping them understand these concepts and their relevance to everyday life” (MAIA *et al.*, 2015, p. 2235, our translation). We understand that it is necessary that Elementary School students expand their knowledge about the Multiplicative Field, and for that teachers who work in that stage of education must understand the specificities of that field and provide systematic work with diverse problems in the classroom.

5 References

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Antonio Luiz de Oliveira Barreto, State University of Ceará (UECE), Education Center, Pedagogy Course

 <http://orcid.org/0000-0001-9871-6338>


Post-doctorate student at the Graduate Program in Education at Federal University of Paraíba (PPGE/UFPB). Doctor on Education from Federal University of Ceará (UFC). Master on Math from UFC. Graduated in Math and Science from UECE. Member of the Research Group Math and Education (MAES) and Project Obeduc. Area coordinator at the Scholarship Program Initiation in Teaching (Pibid/Pedagogy/UECE, 2014-2018). Works as adjunct teacher at UECE, teaching the following disciplines: Math in Early Childhood Education and in Elementary School I and Math in Early Childhood Education and in Elementary School II. At the Pedagogy course, works mainly with the following themes: Early Childhood Education, Math Education, Educational Informatics and Teacher Training.

Authorship contribution: Participated in all stages of creation and development of the article.

Lattes: <http://lattes.cnpq.br/2741856107159943>

E-mail: antonio.barre@uece.br

Rogéria Gaudencio do Rêgo, Federal University of Paraíba (UFPB), Department of Mathematics, Center for Exact Science and Natural Science

 <https://orcid.org/0000-0003-4618-7213>

Baccalaureate in Math from UFPB, Master on Philosophy from UFPB and Doctor on Education from Federal University of Rio Grande do Norte (UFRN). Professor at the Department of Mathematics from the Center for Exact Science and Natural Science at UFPB and professor at the Graduate Program in Education at the Center for Education at UFPB. Has experience in the Education field, with emphasis on Methods and Techniques of Education, working mainly with the following themes: Education Methodologies, Teaching-Learning, Teacher Training and Formation of Concepts in the Math field.

Authorship contribution: Participated in all stages of creation and development of the article.

Lattes: <http://lattes.cnpq.br/3882610066313180>

E-mail: rogeria@mat.ufpb.br

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