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A World-Semantics: An Outline of a Proposal

Uma Semântica de Mundo: um esboço de uma proposta

Konrad UTZ
Doutorado em Filosofia - Universidade de Tübingen.
professor efetivo da Universidade Federal do Ceará e do
PPGFIL UFC.
e-mail: utz@ufc.br

ABSTRACT:
Outside academic philosophical discourse, the World is usually thought of with a capital "W", it is thought of as changing, and as being identical in its changes. The World was different 50 years ago, and it could have been different from what it is now. Most people also assume that the World is undecided as to the future. The possibilities of what may happen are limited, but it is not yet totally defined what the World will be like tomorrow. This article proposes a semantic model of the World in this sense. For this, it combines modality and temporality, introduces the concept of facticity as different from actuality and necessity, and devises the mixed category of an individualistic state of affairs. This may sound complicated, but it turns out that this model fits with a language that is surprisingly simple, using just one additional symbol as compared to standard modal logic – and which is structurally close to normal language.
KEY-WORDS: world, time, modality, facticity, happenstance.

RESUMO: Fora do discurso filosófico acadêmico as pessoas normalmente pensam do Mundo com letra maiúscula. Mais ainda, elas acham que o mundo mude e que ele seja idêntico em suas mudanças. O mundo era diferente 50 anos atrás, e ele poderia ter sido diferente de como ele está agora. Muitas pessoas também assumem que o mundo não seja decidido com respeito a seu futuro. As possibilidades daquilo que pode acontecer são limitadas, mas ainda não é completamente definido como o mundo será amanhã. Este artigo propõe um modelo semântico do mundo neste sentido. Para isso, ele combina modalidade e temporalidade, ele introduz o conceito da facticidade como sendo diferente da atualidade e da necessidade, e concebe da categoria mista de um estado-de-coisas individualístico. Isso pode parecer

complicado, mas este modelo combina com uma linguagem que é supreendentemente simples, usando apenas um símbolo adicional quando comparada à lógica modal comum – e que é estruturalmente parecida com a linguagem comum.

PALAVRAS-CHAVE: mundo, tempo, modalidade, facticidade, acaso.

A World-Semantics: An Outline of a Proposal[i]

Let us start with identity. Let us say that “A” is a statement, and formulate, for any A:

(1) $A = A$

Intuitively, we distinguish between numerical and qualitative identity. Let us stipulate that (1) expresses both: A is *given* by itself (and only by itself); and A (strictly) coincides with itself (and with itself only). Whatever coincides with itself is *consistent* with itself. Let us interpret the following two statements as expressions of these two aspects:

(2) $A \leftrightarrow \neg\neg A$

(3) $A \rightarrow \Diamond A$

Let us formulate that (2) expresses the aspect of *factuality* of A, and (3) the aspect of *possibility*. We see that possibility is implied by factuality, but not vice versa. In this sense, factuality is more basic than possibility. On the other hand, possibility is “wider” than factuality: possibility is (or may be) not only given if factuality is given. A may be possible even though it is not (actually) true.

(2) and (3) imply two fundamental concepts. Both imply logical relationality or inferentiality: even (2), as the more basic, implies that any A has, as such, logical relations, such as, e.g., the relation of equivalence to its double negation, and, with this, the relation of negation to its negation “¬A”. Note that, in this text, I will not treat identity as a relation. However, we need relationality to express the aspects of identity. If we take logical space (LS) to be that within which anything (that is within it) *eo ipso* is in (definite) logical relations, then we can also say: (2) implies LS.

On the other hand, according to (3), ◊A implies something more than just LS. This is because A and ◊A are not equivalent. Hence LS must be larger than the totality of statements. It must be the space of all “statements-at-all”, i.e., independently of their factuality. Let us call a statement-at-all a “proposition (P)”. (“A” *in* “◊A” is a proposition, not a statement; “◊A” means, at this stage: “A is (a P) in LS". Of course, "◊A" as a whole should be a statement – we will come to this.) Hence, LS is the space within which all Ps are in definite relations to one another. However, a proposition A is in definite relations with other Ps by virtue of A = A, otherwise, its relation to ¬A would not be definite, or, in other words: A would not have a *logical* relation to it. But since we defined that, if A = A, then not only ¬¬A is in LS, but also ◊A is in LS, the condition for a P to be in LS is not factuality, but consistency.

With this, we have explained *how* LS is larger than the totality of statements (i.e., all consistent Ps that are not in the scope of a modal operator). However, with this, the totality of statements automatically constitutes a specific region or domain in logical space. It is the domain where, e.g., "A" is *true* or *false*, even though the P "¬A" also may be given in LS, since the condition for being given in LS only is consistency, not truth. We formulate: a statement is true (or false) in virtue of a state of affairs (SOA). There are endless discussions about truth, which I will not go into here. That domain in LS with regard to which non-modal statements are true or false is a totality of SOAs. Intuitively, we call this domain the "world".

As indicated, there is a difference between a statement and an SOA, and there is a difference between a statement A and its "respective" SOA. Again, I will not go into the difficult question of the relation of both. We will simply formulate: "A" *expresses* a SOA; and: "A" is *true in virtue of* the SOA it expresses if that SOA is *given*. Furthermore, as follows from the above, a SOA is given in a world. We allow ourselves to slur the difference between a statement A and its respective SOA with regard to LS and formulate that LS *contains* an SOA if a statement that expresses that SOA is in LS. Hence, LS *contains* the world.

Since anything in logical space must be identical to itself, the world must also be. Hence, it must be consistent with itself, hence, all statements regarding the world must be consistent with one another. With this, we arrive at a *general* definition of "world" as a consistently determined maximal SOA (we leave all subtleties aside) expressed by a consistent maximal P (an MP – a consistent P that contains a complete totality of Ps, i.e., to which no P could be added without turning the result inconsistent). Up to this point, the distinctions we made seem, in principle, inevitable (of course, we may disagree about their exact definition).

However, once we have defined "world" as a universal term, it is theoretically possible that there is more than one instance of a world, i.e., that there are more worlds than only the actual one. Once we have admitted this, LS automatically contains all possible worlds (PW), since LS is the space of all Ps that are consistent (according to the simplification we introduced), including MPs. However, once we have admitted the theoretical possibility of more than one world, all worlds (all that can be a world) are theoretically possible. Hence, for each consistent MP, there is a PW that contains the SOA which makes this MP true. This means, according to (3), that the world is a PW. However, since (3) is not an equivalence, a PW need not be *the* world, i.e., that PW whose partial SOAs make all statements (all consistent Ps that are not in the scope of a modal operator) true or false. Hence, we have to specify which PW is that latter PW. We do this by the term "actual".

Now we can formulate Possible World Semantics (PWS) for modality: "◊A" expresses that *there is* a PW with regard to which "A" is true; and "□A" ("necessarily: A") expresses that "A" is true with regard to all PWs. That is, we can explain modality via truth with recourse to LS and quantification.

Of course, all of this is well known. I explained this step by step to show that all this is "theoretically inevitable" once we accept the notion of LS, together with the distinctions of truth and falsity, of Ps and SOAs, and of factuality and possibility. It is not "theoretically compulsory" to use PWS, but if we start from the conception of LS, MPs are contained in LS; if we form the *concept* of a world, then there can be (in the theoretical sense) different instances of a world, hence there can be different worlds that correspond to (i.e., which make true) MPs in LS, and nothing can forbid us to pick them out and quantify over them. On the other hand, it is "theoretically compulsory", and not only "theoretically possible" to single out the actual world, i.e., the PW with regard to which "A" is true or false (this does not mean that it must be singled out absolutely, it may be singled out only indexically). Otherwise, if "A & ◊¬A" is consistent, the truth of "A" simpliciter, i.e., without a modal operator, would be undecided. That is, we cannot do without *actuality*.

With this, we must specify our definition of "statement": A statement is not just a consistent P that is either true or false, it is a consistent P that is made true or false by the actual world. We can still say that a statement A is true or false *with regard to* a counterfactual PW. But in this case, we speak about the *counterfactually possible* truth of A. A is *simplicter true* only with regard to the actual world. However, we need simpliciter-truth, i.e., we need statements, not only Ps. We can, of course, (eventually) truthfully say "◊A", even if "¬A", i.e.: "A" is counterfactually true with regard to some PW. However, the truth (or falsity) of *this* sentence: "'A' is counterfactually true with regard to some PW", must then be *simpliciter* – that is, this P must be a statement. It cannot be made true (or false) by all worlds, since "true" is defined by reference to exactly one PW – otherwise, *only* modal statements could be true (or false). Hence, the meaning of: "◊A" is true, must be: "'A' is true within some PW" is made true by the actual world. This means, of course, that the world must contain its own relatedness to other PWs. It does not need to contain (and cannot contain) those PWs, but its own relatedness to them in LS. (Developing this further would lead us to the discussion of modal accessibility relations, which I want to avoid.)

This will be important for the following: a world, or at least the actual world, can (and even must) "contain" or "comprise" its relatedness to other Ps in LS, even though it does not contain these Ps (as true) or the respective SOAs (the SOAs that make them true). For this, we expand the definition of "PW": a PW is a maximal SOA together with the MP that expresses this SOA. Note that the respective MP may express more than just this SOA, namely the relatedness of its respective SOA to other PWs, i.e., the possibilities that are accessible from this world. Maybe "contain" is not intuitively the appropriate term to express this, maybe it would be better to say this relatedness is "part of the world" or "belongs to the world". But terminology does not matter. What is important is that, though "$\Diamond$A" does not *express* a SOA *in* the actual world, the actual world makes the *statement* that $\Diamond$A true (or false) – not by a SOA in it, but via its relatedness to Ps (in PWs) outside it. And this is the simpliciter-truth (or simpliciter falsity) of this statement. (Some people may not agree with this, insisting that "$\Diamond$A" expresses an SOA *within* the actual world; however, perhaps they would at least concede that the world contains that SOA *in virtue* of its relatedness to other PWs in LS. This would be sufficient for the following.)

At this point, I want to introduce some new terminology. For reasons that will become clear in the following, I want to call a PW (as we defined it up to now) a "mundus", where the respective adjective is "mundane". Hence, a mundus (M) is a consistently determined maximal SOA together with the MP that expresses that SOA (and, in addition to this, the relatedness of that M to other Ms in LS). The plural of "mundus" is "mundi", but for simplicity, I will write "Ms" for "mundi", not "Mi". I will later loosely reintroduce the term "PW" as the conception of M in classical PWS, without the additional differentiations that I will introduce. I will reserve the term "world" for something completely different, which I will get to at the end of this article.

In normal understanding of language, expressions like "A" are automatically located within LS and within an M (more precisely, within the actual M). But, once we have introduced these two concepts, we can also abstract from them. However, we cannot abstract from both at the same time, since without any logical relations and any mundane location, "A" (as a linguistic expression) cannot have any link to reality or to (the rest of) language. This still leaves us with the possibility to abstract from only one of them at a time. Since the concepts of LS and M are linked to the two aspects of "A = A", we can explicate this possibility of abstraction with regard to each of them.

If we take the aspect of factuality, expressed by "$\neg(\neg A)$", and abstract it from its logical relations, we get "absolute factuality", i.e., factuality not as an aspect of something, but as "simply given factuality". Let us call this a "fact". A fact, as such, is without any (logical) relations. A fact is a simply-given. But how can it be given if it is not given in LS? "Where" can it be given? It certainly cannot be given as a statement (or a P) because a statement is a piece of language, and language is constituted by relations, nor can it be expressed by a statement (at least not directly, see below), since statements express (only) SOAs that are located in LS (see above).

To understand this, we must remember the distinction between statements and SOAs. As we have seen, by definition, for every (nonmodal) statement A in LS, there is a partial SOA in every M in LS with regard to which A is either true or false. Hence, there is no statement A with regard to which LS does not contain a (M which contains a) SOA with regard to which A is true. We can formulate: all statements have a *respective* SOA in LS (some have two in the sense that there is an SOA in LS that makes them true and one that makes them false). And since all statements are in LS (by definition), all these respective SOAs are in LS.

However, this does not imply that all SOAs are contained in LS, i.e., that all SOAs are expressed by (consistent) Ps. From what we have defined until now, it is very well theoretically possible that there are SOAs beyond LS. Of course, with this, we also admit that an M may extend beyond LS, since, by definition, an SOA is given in a M.

A SOA that cannot be expressed by a P (because it is not in LS) can (theoretically possibly) still be "described" or "enunciated" by a P. An example in normal language would be: the fact that Peter was married is *described* by the P: "Peter is married". But that P as such, i.e., in the present tense, does not *express* the fact that Peter *was* married. Being in the present tense, that P has no way to "reach" the fact that Peter *was* married. Hence, we can explain by analogy: the relation of Ps (in LS) to facts (in our model) is analogous to that of sentences in the present tense to SOAs in the past (in normal language).

As we defined LS as the space of logical relations, being beyond LS for an SOA means that it is beyond logical relations or that it is logically irrelative or "absolute". Hence, a fact is a "logically absolute" SOA. Mark that for an SOA to be beyond LS does not mean to be inconsistent. Remember that SOAs are contained in LS only by virtue of the statements that express them. *Ps* are contained in LS by virtue of their being consistent, and Ps are *not* contained in LS (only) if they are inconsistent (this follows intuitively from *ex falso quodlibet*: LS would become indistinct if it contained inconsistent statements). There are no consistent *statements* outside or beyond LS.

However, the same does not apply to SOAs. A SOA can be "consistent (with itself)" without being given in LS. We could even *define* that all SOAs are self-consistent. However, in our model, this is not necessary, it will follow from the "Law of Descendance of Facts" (see below). Without LS, the consistency of an SOA cannot be *expressed*. However, SOAs are independent of their being expressed, hence, they may also be consistent without their consistency being expressed (the SOA that a certain SOA is consistent need not be expressed). However, this point does not matter here.

For an M to contain facts, we must modify the definition of "M" slightly. First, note that facts are not inconsistent with anything else since they have no logical relations at all. Hence, an M can be a totality not only of actual SOAs, but also of factual SOAs. It can be "more-than-complete" without being inconsistent. This is because an M is not, as such, an LS. Hence, SOAs in it are not logically related by virtue of being in an M. Hence, it is *not* true of M that: "A & B", where "A" expresses the respective consistent maximal SOA and "B" expresses an additional SOA (i.e., one that is inconsistent with A), since "&" expresses a relation (in the wider sense we apply here), and facts are irrelational. Furthermore, as already said, no P expresses a fact – at least not directly (see below). Because facts fall outside the space of conceptual relatedness, i.e., beyond LS. Consequently, facts do not make the M to which they belong inconsistent, since they simply do not relate (logically) to the (maximal) SOA that is the "core" of M. Hence, we can say that M is a set of a consistent maximal SOA (together with the MP that expresses it) and (possibly) other SOAs (that are irrelational facts, beyond LS). However, we can still define: A M is a complete, consistent set of SOAs, where "complete" means: containing a maximum. Hence, an M contains a maximum of consistent SOAs (i.e., that are "positively consistent" with one another, i.e., that are consistently *logically related* to one another, in LS). Hover, this does not rule out that it contains SOAs besides that maximum of *consistent* SOAs, i.e., "complete" does not mean "exactly complete", it does not exclude "transcomplete". We could formulate: "transcompleteness" in this sense is different from "overcompleteness" in the logical sense, which is tantamount to inconsistency.

With this, M is characterized by a core that is a maximal SOA, together with the MP that expresses it in LS. However, in addition, an M contains Ps that express its relatedness in LS, i.e., modal Ps; and it may contain SOAs beyond LS, i.e., facts. Of course, the question is: what are these factual SOAs that an M contains (besides its actual core SOA)? It would be very inconvenient if they were *all* facts. We will come back to this later; at the moment, we only state that an M *may* (theoretically possibly) contain facts.

Now the question is: how can we talk about facts? We can only talk in language, and language works only within LS. However, there is a possibility for language to reach out of itself, and this is by indexical terms. We can point to something and say: "This is a dog." We can also point to a location and say: "There is a dog". We can further specify that location: "There is a dog in this room". And we can specify it negatively: "There is a dog outside the house". (And we colloquially use the term "beyond" if that outside is, in some relevant sense, discontinuous with the inside.) Analogously, we cannot (logically) refer to facts, but we can indexically *point* to them. More precisely, we can point to LS and indicate facts as that which is beyond LS. Furthermore, we can indicate that there is a certain fact within that realm of our M that contains facts, i.e., the "trans-logical" realm of our M. For this, that realm need not be logically accessible, neither in the sense that something in it is logically accessible nor in the sense that it is logically accessible as such, i.e., as another part of our W. It is sufficient that *LS* is accessible. Then we can point to that "other realm" simply as to *the Beyond* of LS, i.e., by *pointing beyond* LS. Hence, we can say, e.g.: There is a that-A beyond LS. We express this more elegantly by saying: that-A is a fact (in our M, but this is implied simply by that statement being non-modal). Note that "that-A is a fact" expresses an *actual* SOA, i.e., *within* LS. This expression points *from within* LS to something in (the actual) M beyond LS, namely to the fact that A. Of course, "that-A is a fact" expresses a different SOA than is expressed by "A". "that-A is a fact" does not only say that A, but says something about "that A": that that-A is a fact. Hence, it may very well be that (regarding a M) "that-A is a fact" is true, but "A" is not true.

To make our explanations more simple and more intuitive, we introduce a new term for the Beyond of LS, i.e., for the realm of facts (we should not call this realm a "space", since a space is something in which things automatically are related to one another; there are no two spaces, that of logic and that of facts, there is only one space – and its Beyond): remembering the above terminology, we call it "the past"; and we allow ourselves to express "that-A is a fact" colloquially by stating "A" in the past tense: "Peter was married" means "it is a fact that Peter is married" (we see that the terminology I introduced here is *not* intuitive in this case, i.e., in the case of facts that are not presently actual *any more*: intuitively, we say: "it is a fact that Peter *was* married"). However, at this point, this is a mere terminological convention, "the past" means nothing more than "the Beyond of LS" or "the realm of facts". Furthermore, we introduce a formal notation for "that-A is a fact":

(4) A∫

Visually-metaphorically, (4) "A∫" expresses: inside the (respective) M, beyond LS: A. We call "∫" the "facticity stroke" or "time stroke". Note that we cannot negate "∫". "¬A∫" does mean: "that not-A is a fact", but not: "A is not-factual". The latter would be "A¬∫", but no negation may occur (directly) after a proposition, "A¬∫" is not a well-formed expression. If there was a (internal) negation of facts, then a fact would have to imply the negation of its negation, i.e., it would have to have logical relations. And then there would be, once again, a maximum of facts, since, if that-A would *not* be a fact in M, then "it is a not-fact that A" would be a fact in M. Hence, the realm of facts is not a *space*, also in the sense that we cannot quantify over it *as such*. However, we can quantify over the facts that an M (in fact) contains. Such quantification only comprises those facts that are "positively given" in M (that is, of course, a tautology, but it may help to clarify things). It does not comprise the "absence" of facts in M. Note that, in this sense, "¬A∫" also expresses a positively given (negative) fact – as different from "A¬∫".

We *can* say: ¬(A∫). It is difficult in colloquial language to distinguish this expression from the one we just ruled out. But the difference in meaning is: ¬(A∫) states that it is *not* the case that "A∫" expresses a fact; whereas "A¬∫" would (positively) express the *absence* of a fact – and that cannot be done. Of course, if the presence of something in some *space* is negated, its absence is automatically affirmed. But the realm of facts is not a space.

Now we go to the aspect of possibility implied in "A = A": ◊A. If we abstract from the localization of that possibility in an M, we get the pure or absolute possibility of A in LS. So to speak, we establish a region of LS where Ps are "not in contact" with SOAs, i.e., where it is undecided whether they are actual or not – and, consequently, if they are possible or not in the sense of *actually* being true *in some M*. At this stage, this leads to an awkward duplication: MPs are contained twice in LS, once as part of an M, i.e., together with the maximal SOA they express, and once isolated on their own. However, later we will be able to dispense with all Ms except the actual M; and the MP expressing the maximal SOA of the actual M need not be given only abstractly in LS.

Of course, that region of absolute Ps may (or, more exactly, must) contain *any* A of which: A = A, i.e., any consistent A (including any consistent maximal A, but without its actuality/non-actuality, and hence, without possibility-relative-to-actuality). Hence, there is no way to express necessity with regard to that region via quantification, anything in it is a pure or absolute possibility.

To *define* LS in this sense, i.e., we only need to define: LS is the space of all consistent Ps (i.e., that within which these Ps are *eo ipso* related to one another) – instead of: LS is the space of all consistent maximal SOAs (with the implied understanding that one of them is actual, and/or that there are relations of modal accessibility between them). The simple-possibility of a P is just its being in LS. Its "simple-necessity" would be the absence of its negation in LS.

Since "absolutely possible" simply means: given extramundanely in LS, we need only one symbol to notate it formally:

(5) ∫A

Visually-metaphorically, (4) "∫A" expresses: inside LS, outside any M: A. Note that "A" may very well be a maximal P (MP); but in "∫A", "A" does not occur as expressing any SOAs, i.e., as "being together" with the SOAs it expresses. This being-together would (exactly) be an M, different from an MP. We can explain: the facticity stroke left of a P places this P outside any M (outside the factual).

Since "ʃA" is within LS, it may very well be connected to Ps that express SOAs *within* an M by logical relations. More exactly, "ʃA" may be either compatible or incompatible with a M. As statements by default speak about the actual world, "◊ʃA" expresses: "A is compatible with the actual M", or "A is *possible in consideration of* the actual M"; and "¬◊ʃA" expresses: "A is incompatible with the actual world", or "A is *impossible in consideration of* the actual M". Likewise, "¬◊¬ʃA" expresses: "non-A is impossible in consideration of the actual M", or "A is necessary in consideration of the actual M", i.e., "□ʃA"; and "◊¬ʃA" expresses: "non-A is possible in consideration of the actual M", or "A is non-necessary in consideration of the actual M", i.e., "¬□ʃA".

E.g., if in the actual M Peter is married, then it is possible in consideration of the actual M that Peter is not married because Peter's being married in the actual world is compatible with his being unmarried outside it. On the other hand, if in the actual M: 2 + 3 = 5, then it is impossible in consideration of the actual M that not: 2 + 3 = 5 since this is not possible outside it.

Up to this point, Possibility-in-consideration-of-M is not very interesting, because "□ʃA" amounts to "□A" and "◊ʃA" amounts to "◊A". However, based on what we have developed up to this point, we can define the Principle of Future Facticity:

(6) A → □ʃAʃ

Or, in colloquial terms: If A, then it is necessary in consideration of the actual M that that-A is a fact. Or even more colloquially: … it is necessary in consideration of the actual M that A was the case. With this, if in M: A, then it is necessary in consideration of M: Aʃ, i.e., M is *only* compatible with MPs that contain "Aʃ", and with Ps that are compatible with "Aʃ".

As it should already have become clear where this is heading, I introduce the following colloquial terminology: For "it is necessary in consideration of", we may say: "it will (inevitably) be the case in the future of". That is, if we speak in the future tense or express in some other way that we talk about the future, by default, we express: □ʃA – or the other way around: I will, in this text, express "□ʃA" informally simply by using future tense or by saying "in the future A" or "*inevitably* (in the future) A"; different from "□A" which, of course, is expressed colloquially by "necessarily A", "A must be the case" and the like. Analogously, if I want to express informally "◊ʃA", different from "possibly A (◊A)", I will say: "*maybe* A (in the future)".

Hence, "A → □ʃAʃ" means: If A, then it will be the case that A is a fact in the future of A; or simpler: If (now) A, then it will have been the case that A.

Remember that the SOA that-A is different from the SOA that it is a fact that-A, hence, it is *not* true: A → A∫. In the terminology we established above, we can say: e.g., "Peter is married" does not imply "Peter was married in the past". But: If A, then necessarily in consideration of the M of A: that-A is a fact. Remember that in *this* case this latter P, i.e., the P "that-A is a fact", is not given *within* M, but outside M (but inside LS), it is a transmundane P. That that-A is a fact may be given in *other* Ms, but "□∫A∫" does not refer to the SOA 'that it is a fact that A' in other Ms, but it states that P *simpliciter*, in abstraction from any M or "outside" M – however, *in consideration of* the actual M. Hence, "□∫A∫" does *not* express: necessarily that-A is a fact, i.e., not: □A∫ (A is a fact in all Ms). It expresses that "A∫" is necessary *in consideration of* the actual M. E.g.: If: Peter is married, then: necessarily in consideration of M: it is a fact that Peter is married; or alternatively, simply: Peter will have been married.

From (6) follows:

(7) A∫ → □∫(A∫)∫

However: (A∫)∫ → A∫, since: if it is a fact that that-A is a fact, then it is simply a fact that-A. Hence, simply:

(7*) A∫ → □∫A∫

Now we go back to the question: What are the facts that a certain M contains? As facts are not restricted by any logical relations, it is completely arbitrary what is a fact in an M. Of course, this is unsatisfactory. But now we can formulate a theoretically possible restriction on the facts of an M. If there is an M: א, and an M or MP (from now on I will use "MM" as a superordinate concept comprising "M" and "MP"), i.e., a MM ב that (actually) contains all those Ps that are necessary in consideration of a MM א, then ב is *factually compatible* with א. That is:

(a) (□∫A in א) → (A in *every* ב)

(b) (□∫A∫ in א) → (A∫ in *every* ב)

We also call ב a "successor" of א: א ◄ ב. Note that succession of MMs is *generally* defined, independently of any reference to an actual M. Note also that, at this stage, ב may contain facts in addition to those it must have in virtue of its compatibility with א (We cannot simply formulate: that contains more facts than the other, since an M or MP may contain infinite facts, and in this case, the number of facts contained in M does not increase by additional facts.) Note, finally, that according to this terminology, a SOA in ב may be necessary or possible in consideration of א. That is, not only (a) and (b), but also:

(c) (◊∫A in א) → (A in *some* ב: א ◄ ב)

(d) (◊∫A∫ in א) → (A∫ in *some* ב: א ◄ ב)

With this, factual compatibility imposes a *restriction* on the Ps indicating facts (Ps of the form "A∫", i.e., "factic propositions") of ב in the sense that ב must contain (cannot not contain) certain such propositions (for simplification, I will say that an MP "contains a fact" if it contains a respective factic proposition – even though, strictly speaking, this is wrong, since MPs do not contain SOAs, since they are not Ms). Of course, this leaves open what other facts ב contains. However, we can introduce the Law of Descendance of Facts (LDF): a MM can contain a fact *only* by virtue of being a successor, i.e.:

(e) (A∫ in ב) → (A in א: א ◄ ב)

With this, at this point, it is still (theoretically) possible that there are Ms that are not successors ("primary" Ms, or "bereshit-Ms"), but they do not contain any facts.

However, the expressions (a) to (e) are not formulas of a formal language, since they are metalogical expressions, because Ms cannot be named individually in LS, only the actual M can be singled out by its actuality. Hence, the concept of factually compatible worlds is not a logical concept. But it is one we intuitively understand and that we can use to explain things.

We define further: two Ms that have all their facts in common are "factually congruent". Then we can define: if two MMs ב and ג are both factually compatible with א, and ב and ג are factually congruent, then ב and ג are "factually co-compatible" with א, or they are "strictly simultaneous successors" of א. We can call that in which ב and ג are strictly simultaneous their "stage of succession". A stage of succession as such, considered in itself, may also be called a "future stage". Once we have defined this, we can define: two MMs are "(simply) simultaneous" if they share their state of succession. They do this if they are factually congruent at *some* stage of their past and factually congruent with regard to *all* (non-future) facts they contain that were inevitable at that past stage.

Since we have differentiated stages of succession, and, with this, successors of successors, the expression "□∫A" cannot mean anymore: *simply* necessary in consideration of the actual M. It now must mean: at *some* future stage necessary in consideration of the actual M. Hence, it seems reasonable to introduce the expression: at *all* future stages up to *some* future stage necessary in consideration of the actual M: A; or: *continuously* up to some future stage: A, or, colloquially: until sometime (s.t.): A. We write:

(8) □∫∫A

Consequently,

(9) $\Box\int\neg\int A$

means: continuously *not* (only) until s.t.: A; or: continuously unlimited: A; or: at all future times: A; or colloquially: forever A. (This may not seem intuitive at first, but it will prove to be consistent in the following.) Now we can formulate: "…$\int$…" quantifies over the successors of (exactly) one line of succession or one *possible future course*.

This is the meaning "$\int$" has, *on the one hand*, as being left of a P. On the other hand, however, as being on the right of a modal operator, it is an index to that operator and *qualifies* the respective necessity/possibility as "necessity/possibility *in consideration of*", or as "*factually conditioned* necessity/possibility"[ii], i.e., as quantifying not over the totality of Ps or of MMs or PWs, but over the totality of the lines of succession of an M that is factual relative to all these lines, as being that to which they succeed. By default, that M is the actual M, but in "$\Box\int(\Box\iint A)$", e.g., "$\Box\iint$" quantifies over the lines of succession to some successor to the actual M.

Hence, the logic of factually conditioned necessity is the following:

(10) $\Box\int\neg\int A \leftrightarrow \Box\neg\int\neg A$ (extraction of internal negation)

(11) $\Box\int\neg\int A \rightarrow \Box\iint A \rightarrow \Box\int A$

(12) $\Box\neg\int\neg\int\neg A \leftrightarrow \Box\int A$

(13) $\Box\int\neg\int\neg A \rightarrow \Box\iint\neg A \rightarrow \Box\int\neg A$

(14) $\Box\iint A \leftrightarrow \Box\int\neg\neg\int A \leftrightarrow \Box\neg\iint\neg A$ (symmetrical extraction of negation)

The last equivalence of (14) is only valid if there are no direct successors, i.e., if we accept the geometrical 2^{nd} Axiom of Order for succession of MMs. (14) can be seen as a reason why we should accept this axiom. We cannot define "$\Box\iint A$" by some equivalence to "$\Box\int$" together with negation like we can with "$\Box\int\neg\int A$". But since we can say:

(15) $\Box\int(\Box\int\neg\int\neg A) \rightarrow \Diamond\iint A$

If at s.t.: necessarily nevermore A, then possibly up to s.t. A.

(16) $\Box\iint A \rightarrow \Box\int(\Diamond\int\neg\int\neg A))$

If until s.t. A, then necessarily from s.t. on possibly nevermore A.

That is, "…$\iint A$" can be defined as that whose possibilitation "$\Diamond\iint A$" is the *maximal contrary possibility* that "$\Box\int(\Box\int\neg\int\neg A)$" implies (the latter also implies "$\Diamond\int A$", but that is not the *maximal* contrary possibility it implies, since: $\Diamond\iint A \rightarrow \Diamond\int A$, but not: $\Diamond\int A \rightarrow \Diamond\iint A$) and whose neccessitation "$\Box\iint A$" implies "$\Box\int(\Diamond\int\neg\int\neg A)$" as its maximal contrary possibility. ("$\Box\int A$" also implies "$\Box\int(\Diamond\int\neg\int\neg A)$", since there is no last point in time, but *maximally* implies "◊∬¬A & □∫(◊∫¬∫¬A)".) We could say that "…∫…" and "…∫¬∫…" in "□∫(□∫¬∫¬A)" and "□∫(◊∫¬∫¬A)" together *contour* "…∬…". "◊∬A" is the space left free in the future by "□∫(□∫¬∫¬A)" (and conversely correspondently). Interestingly, "…∫…" and "…∫¬∫…" combined in the inverse order, "□∫¬∫ (□∫¬A)", "in the future time and again not A", implies the maximal contrary possibility "□∫¬∫ (□∫ (◊∬A))", "in the future time and again possibly for s.t. A".

This is why the temporal aspect of our semantical model only works in combination with its modal aspect: without this, we could not define "…∬…". But without "…∬…", our language would be incomplete.

The logic of "∫" and "∬" works normally with modal logic, e.g.:

(17) $\Box\!\int\!\neg\!\int\! A \leftrightarrow \Box\neg\!\int\!\neg A \leftrightarrow \neg\Diamond\neg\neg\!\int\!\neg A \leftrightarrow \neg\Diamond\!\int\!\neg A$

(18) $\Box\neg\!\iint\! A \leftrightarrow \neg\Diamond\neg\neg\!\iint\! A \leftrightarrow \neg\Diamond\!\iint\! A \leftrightarrow \Box\neg\neg\!\iint\!\neg A \leftrightarrow \Box\!\iint\!\neg A$

Expanding our language to "∬" and "∫¬∫" is not only reasonable but also indispensable if we want to sustain our model in view of the plurality of future stages, for then (6): A → □∫A∫, is not sufficient, we must stipulate:

(19) $A \rightarrow \Box\!\int\!\neg\!\int\! A\!\int$

However, up to this point, we have not yet found a way to express: *simply* always, and: *simply* eventually, but only: in the future always/eventually. Note that we can say: necessarily in the future eventually (i.e., "eventually" does not mean "possibly" according to this terminology).

According to (9), if in an M א. □∫A; then there may be successors to this M in which: A & □∫A; and (eventually) others in which only: □∫A; and others in which: A∫. Eventually, there even are some in which: A∫ & ¬◊∫A; since "□∫A" means: (at least) once in the future: A. However, in these last two cases, the MM in question cannot be a direct successor to א, there must be an intermediate successor in which: A. If this is granted (see above), then all but the last of the above occur in *some* successor of א. All these successors may even be simultaneous, since, obviously, "□∫A" does not establish when exactly A will be the case. In fact, up to now, the expression "when exactly" does not make sense on the basis of our model. Stages are fixed not abstractly and absolutely, on a timeline, but only relative to an MM (by default: the actual M) and its successors; nor have they absolute distances to one another, they can only be ordered by their successor relations. E.g., if there is an M א, and there is a successor ב of א, and there is a successor ג of א, of which ב is a successor, then we can say colloquially that there is a time in between א and ב. However, we do not know how "long" this time lasts, our model does not provide a concept of duration. That is, this model is a *logical* model, not a mathematical one – there are no distances (in the strict sense) in LS; which, on the one hand, seems to be an advantage of this model; on the other hand, it shows the limitations not so much of this model as of logic versus mathematics.

Of course, it follows from (9) that:

(20) A → □ʃ(□ʃ¬ʃAʃ)ʃ

But this shows that, naturally, SOAs of factually conditioned necessity can also be facts since (20) implies that there is an MM that is a successor to the actual M in which: (□ʃ¬ʃAʃ)ʃ. This means that we can express past factually conditioned necessity in our model: we can, e.g., say that something was necessary or possible yesterday. With this, we can establish an order of succession between facts, i.e., we can differentiate "past stages" with regard to the facts that an M א has in virtue of being the successor of another M ב, i.e., in virtue of (9) regarding ב.

Hence, "Aʃ" should now mean: "that A is a fact (at *some* past stage of the actual world)", and "Aʃʃ" should mean: "that A is a fact continuously up to some past stage (of the actual world)", that is: "since some time: A", and "Aʃ¬ʃ" means: "that A is a fact continuously unlimited down the past stages of the actual world" (remember that we quantify only over the factual stages of the actual world, not over some universal "space of facts", which does not exist). The logic of "ʃ" and "ʃʃ" and their negations is already established by the logic of factually conditioned necessity, i.e., their meaning *is* established in LS. Hence, we can use "ʃ", "ʃʃ" and "ʃ¬ʃ" to point (from *within* LS) at something beyond LS without fear of inconsistency or "meaninglessness" of these terms.

"ʃ" right of a P is not really an operator (which intuitively should always be to the left or in between Ps), it is an "indicator": it indicates the SOA that P describes as being a fact (of the respective M), i.e., as being beyond LS. And since, according to LDF, all facts of an M must descend from a predecessor to that M, "ʃ" may quantify that descendance: "Aʃ" indicates: is a fact descendent from *some* predecessor (which is a tautology according to LDF – which shows that quantification is secondary in this case); "Aʃʃ": is a fact descendent from all predecessors down to some predecessor; and "Aʃ¬ʃ": is a fact descendent from all predecessors (down to not only some predecessor). That is, strictly speaking, "ʃ" does not quantify *over predecessors*, but "adverbially" quantifies descendance (which latter is implicit in facticity, according to LDF), regarding its provenance. "Aʃ" cannot just mean: "A (is actual) in some predecessor", since we need facticity to define predecessors (that is how we started). Hence, we cannot *define* facticity with recourse to predecessors. So "Aʃ" must *originally* indicate A as a fact (of the *actual* M), and only secondarily quantify its descendance (from *another* M). I.e., descendance does *not* define facts. It is the other way around: facticity is needed to define factual descendance.

We define a (past) stage of a factual SOA as the stage of a predecessor-M in which that SOA is (or, in colloquial terms: was) present. (Of course, an SOA may have been actual in different Ms that are successors one to another.) With this, facts are *indirectly* ordered into past stages, via their descendance – they cannot be ordered directly.

Now let us look back at (7): Aʃ → □ʃ(Aʃ)ʃ. According to what we have developed now, it still holds that: (Aʃ)ʃ → Aʃ, but the antecedent and the consequent do not mean the same thing. "Aʃ" means that "A" is a fact descendant from a predecessor to the actual world, whereas "(Aʃ)ʃ" means that "A" is a fact descendant from a predecessor to a predecessor to the actual world. However, since successorship does not imply any fixed or minimal temporal distance (see above), "(Aʃ)ʃ" as such is "informationally equivalent" to "Aʃ". I do not "know more" from "(Aʃ)ʃ" than from "Aʃ", they both inform only that A was the case at some stage in the past. However, this changes if we say, for instance: (A & Bʃ)ʃ – e.g.: When Socrates was born, the pyramids had (already) been built. That is, such expressions serve to establish a relative successional order of facts (colloquially: a temporal order of past SOAs). Note that when: (A & Bʃ)ʃ, then also *simply*: Bʃ – the pyramids were built.

Of course, we can also formulate: When the clock in Greenwich showed the end of July 28, 1914, on the Gregorian calendar, WWI had (already) begun. However, in the context of our model, "when the clock in Greenwich showed the end of July 28, 1914, on the Gregorian calendar," expresses a concrete SOA. It does not fix a past stage abstractly on a "timeline" (see above). Our model does not require such a timeline. Nor can we project that model directly on a timeline. We can do so only indirectly through statements expressing concrete SOAs. Our model does not have an "inbuilt clock" – which seems intuitive to me.

This leaves us, once again, with the question: is only "(Aʃ)ʃ → Aʃ" valid, or is "(Aʃ)ʃ ← Aʃ" also valid? The latter would express that between a predecessor and a successor, there is always a third successor/predecessor. This seems intuitive, but our model does not require us to introduce that axiom. Note that if we want to introduce it, we do not need to introduce it for both the future and the past, we could introduce it, e.g., for the future only.

With the vocabulary we have developed up to this point, we can already do a lot of useful things. We can express, e.g.: until now (always) A: Aʃ¬ʃ; from now on (always) A: □ʃ¬ʃA; from now on maybe always A: ◊ʃ¬ʃA; (at least) once upon a time A: Aʃ; sometimes (in the future): □ʃA; maybe sometimes (in the future): ◊ʃA; from some future point in time on always: □ʃ(□ʃ¬ʃA); maybe, from some future point in time on always: ◇ʃ(□ʃ¬ʃA); since some point in the past inevitably eventually A: (□ʃA)ʃ – note that this does not say whether A has already happened or not; since a certain point in time always: (□ʃ¬ʃA)ʃ).

We could further introduce an operator "@" that limits the scope of the facticity bar to the actual stage of the respective M. Hence, "A@ʃ" and "□ʃ@A" would make no sense since they would mean: "has happened sometime from now on" and "will happen sometime before now. But it makes complete sense to formulate: "□ʃA@ʃ", "will eventually have been a fact sometime from now on"; and "□ʃA@ʃ¬ʃ", "will have been a fact always from now on", i.e., "from now on until sometime"; or "(□ʃ¬ʃ@A)ʃ", "at some time in the past: necessarily until now", i.e.: "since some time inevitably until now". But "@" is problematic since it is an M-indexical term, i.e., it has a different meaning in every world. Hence, any P containing "@" in an M could not be necessarily contained in its successor Ms, since it would have a different meaning in these Ms. However, it seems to me that we do not "need," in a logical sense, to introduce "@", since the succession of past and future stages of an M is clearly defined without it.

Still, up to this point, our model does not seem to take us very far, because a successor to an M, up to now, may still *actually* contain all logically possible SOAs, only the SOAs it *factually* contains are restricted by its successorship. But now we can change this. Because now we can express: (□ʃ¬ʃA)ʃ¬ʃ, since ever forever A, or: (simply and strictly) always A. Hence, we can formulate: (□ʃ¬ʃ(A → □ʃB))ʃ¬ʃ, strictly always: A implies inevitably eventually B. An example would be: strictly always: (x is a human being) implies inevitably eventually (x dies); and we can formulate, e.g.: Always, after a period of A, a period of B (□ʃ¬ʃ(Aʃʃ → □ʃʃB))ʃ¬ʃ, ("always sunshine after the rain"); or: (□ʃ¬ʃ(A → □ʃ¬ʃB))ʃ¬ʃ, strictly always: A implies inevitably forever B. An example would be: (x is dead) implies inevitably forever (x is dead): (□ʃ¬ʃ(A → □ʃ¬ʃA))ʃ¬ʃ. With this, an M ℵ can (theoretically) impose restrictions on its successors that go beyond logical restrictions and the Principle of Future Facticity. Successors can now be conditioned by laws of ℵ, i.e., laws that are specific to ℵ (and that, in turn, specify ℵ). Let us formulate: successors may be *nomologically* restricted by the M they succeed.

Note that expressions of the type "A → □ʃB" do not formulate, as such, natural laws. Such laws are much more general on the one hand and (usually) more precise on the other. And they are expressed formally (typically) by mathematical equations, not by logical implications. However, "A → □ʃB" (typically) expresses a temporal consequence that is *based on* natural laws (though it may also express consequences based on logical laws, such as: "(A = A) → □∫¬∫(A = A)"). E.g., it seems obvious that "(x is a human being) implies (inevitably eventually x dies)" is true *in virtue of* or *based on* certain natural laws. I will not discuss here how "in virtue of" or "based on" should be defined exactly in this context. It seems to me that this is a difficult question since it implies the question of how we can get from mathematical equations to something very different from such equations, namely, to temporal consequence. And it is a question that would take us well beyond our present task of formulating a World-model. For the plausibility of that model, it is sufficient that, intuitively, there is something that "supports" SOAs of the type "A → □∫B" in our world, and that we normally call "natural laws".

With nomological restrictions on successors, we can drastically reduce their extent (though there will still be infinitely many). We can even (theoretically) reduce them to exactly one successor at each stage, i.e., we can devise deterministic Ms. Since we can do so, our model, up to now, automatically comprises such Ms. However, our model would not make much sense if there were *only* deterministic Ms, and it would not make much sense for understanding *our* world if our world were deterministic. Hence, we will stipulate the following: there are nondeterministic worlds – which seems inevitable in our model, since nomological restriction only occurs within Ms, hence, certain restrictions may occur in some Ms and in others not.

Now, we come back to identity. Since Ms extend beyond LS into what we call "the realm of facticity" or "the past" (but what is really nothing else but the Beyond of LS), we must extend the validity of "A = A" beyond Ls. Identity in this sense must be "translogical". If we consider *individuals*, then we can say: "a = a" is true in all LS and beyond, i.e., I am identical to myself not only in all PWs (as in traditional PWS), but also all in Ms *in their respective totality*, i.e., including their facts. That is, in colloquial terminology, individuals are identical to themselves in their past and even in their possible past (I could have studied mathematics instead of philosophy).

However, a problem arises from this if we accept Leibniz's nonidentity of discernibles. In traditional PWS the problem arises of how I can be identical with myself if, in one PW, I am a born Brazilian, but in another PW, I am not a born Brazilian (but a born Chinese, e.g.). The answer is, of course: we include possible predications into the set of those predications that an individual carries as such. That is, it is "part of my identity" that I could have been born Chinese. That is, my identity extends over the whole of LS, I am not in a *relation* of identity to me in other possible worlds, but the I that I am extends into these worlds. From this arises the problem that I am, *in fact*, that I that is born Chinese, but in another sense, I cannot be that I. On the other hand, I am, in fact, a fascist genocidal dictator (in some

PW), I am that very individual that is a fascist genocidal dictator. But I do not *identify* as a fascist, let alone a genocidal dictator. Hence, there seem to be different understandings of "identity" in normal language that are not only due to its vagueness and its use of metaphors but are of utmost existential importance to us.

We can understand some of these differences in our model, where we consider Ms instead of PWs. I am identical to myself not only in all MMs (including their facts) and in all possible SOAs, but *specifically* in all successors to the actual M. The specificity of the resulting possibilities is that I *can actually be* (in the future) what I am in these Ms, since these MMs are possible *in consideration of* the actual M.

We easily see that this specificity is existentially crucial. I am not very concerned about the fact that there is some MM where I am tortured to death, i.e., that this is a logical possibility. According to what we discussed above, it is really me who is, in fact, being tortured to death. Still, this does not concern me. However, I am highly concerned if I am *actually* imprisoned by a fascist genocidal dictator and I may be tortured to death tomorrow. That is, I am concerned about myself in nonactual MMs if these MMs are successors to the actual M, i.e., if SOAs in these MMs are possible in consideration of the actual M. (Of course, questions of probability also come into play here.) Furthermore, I am, to a lesser extent, concerned about myself in (counterfactual) successors of predecessors of the actual M. I am *relieved* that I was not tortured to death by the fascist dictator because the Revolutionary Army liberated me in time. And I am *angry* that I did not become a millionaire because I sold my Apple shares in 1999.

Moreover, I am (normally) deeply concerned with myself in the *factual* SOAs of the actual M in which I participate, and hence: with myself in all predecessors of the actual M – I am concerned with my past. I am concerned, e.g., with the fact that I insulted my friend yesterday, i.e., in an M that is a predecessor to the actual M; whereas I am unconcerned with the fact that I have committed genocide as a fascist dictator in *some* MM.

Now we go on to the identity of SOAs. Intuitively, SOAs, just like Ps, are just what they are, they cannot be any different. Of course, this is also true for individuals *as such*: an individual cannot be a different *individual*. But individuals are *what* they are in the predicative sense, i.e., they are *discernible* only by virtue of the SOAs in which they participate, purely as such, they are ineffable (indiscernible). And since an individual can participate in different SOAs in different MMs, it can be different or could have been different. As explained above, this has a precise sense, even if we take the discernibility of an individual to include its predication not only in the actual M, but in all possible MMs, *if* we take actuality to be essential for an individual. Moreover, if we take specifically factually conditioned possibility into account, individuals can *change*, i.e., this expression has a well-defined meaning in our model. Contrary to that, SOAs *cannot* change and cannot be different in precisely that sense in which individuals can be.

However, given the concept of "individual" and its various aspects of identity and the concept of "SOA" and its aspects of identity, we can define a new concept of entities (in the widest sense) in our model. I.e., the conceptual elements are given in our model, hence, the (theoretical) possibility of their combination is also given – such a concept is a theoretical *option* in that model.

We can form the concept of an "individualistic SOA (ISOA)", that is, of an SOA that behaves like an individual, an "SOA that changes" and that "could have been different." Of course, this is not an SOA anymore, we are not specifying an SOA by a concept, we are formulating a new concept. An example of such an ISOA is: my life. My life may change, and it could have been different. But, most importantly, my life is going on. An ISOA may be "unfinished" or "incomplete" in the sense that new facts may accrue to it. This is how we would intuitively say it. However, according to our model, it would be more precise to say: an ISOA may (theoretically) be overcomplete. Because an ISOA can only change if it *actually can* be different, i.e., if it *actually* is a partial SOA of this ISOA that a certain counterfactual P is factually conditioned possible (i.e., if "¬A & ◊ʃA" is true regarding that ISOA). But once ISOAs may contain factually conditioned possibilities, they may, of course, contain mutually exclusive such possibilities – e.g., my life may contain the possibility that I travel to Patagonia, as well as the possibility that I do not travel to Patagonia. Colloquially, we say, e.g.: My life may (or may not) take that turn.

The analogous is, of course, true for any M and even for any PW in traditional PWS: any PW "involves" all PWs in the sense that they are "accessible" from it, i.e., that modal statements are true *in* that PW. (That is the normal intuition – as already said, I will not discuss modal accessibility relations here.) Of course, these PWs are not overcomplete in virtue of "involving" these possibilities, since these later are neatly separated from them as *actually* existing *not* in the respective PW but in another PW.

However, different from an SOA, a PW, or an MM, an ISOA may change, i.e., it may *itself* become what, *actually*, is (only) a possible MP regarding it. I.e., it may eventually be *actually identical* with an M which that possible MP describes. But this means that it is *now* possibly actually identical to mutually exclusive Ms. If I should travel to Patagonia, it will have been part of my life that I have traveled to Patagonia. If I do not travel there, it will have been part of my life that I have not traveled to Patagonia. In either case, this will have been my life, i.e., that very life that is my life now – and not only my life in some counterfactual M.

This (theoretically possible) overcompleteness of an ISOA is interesting in that it is not just factual transcompleteness, as in the case of M, where an M may (theoretically possibly) contain facts in addition to the consistently determined maximal SOA that constitutes this M in LS. The possible overcompleteness of an ISOA is overcompleteness in LS. However, it does not make the respective ISOA inconsistent since it is not *actual* overcompleteness – we could call it "hypercompleteness". On the other hand, it does make the respective ISOA "nearly inconsistent" since it is *possibly identical* to mutually exclusive SOAs.

With this, we have defined what this "near inconsistency" really is: it is "undecided identity". It is *undecided* if my life is the life where I travel to Patagonia or where I do not travel to Patagonia. It is not the case, as in traditional PWS, that there are two possible lives of mine, one where I travel to Patagonia and one where I do not – and the "undecidedness" is only that I do not yet know which life is my actual life (i.e., in traditional PWS, I will *not* decide to travel to Patagonia or not to travel there, I will only *find out* what is the case in my life). According to our model, my life (in the example) is *really actually* undecided with regard to my journey to Patagonia. And this yields the concept of "possible identity" – or, the other way around: "contingent identity" expresses the (possible) undecidedness of an ISOA regarding the future.

Regarding individuals, we can now formulate, e.g.: I am (simply) identical to myself as traveling to Patagonia (in some M) and to myself as not traveling to Patagonia (in some other M). But: I am *possibly actually* identical to myself as traveling to Patagonia; and I am *possibly actually* identical to myself or to myself as not traveling to Patagonia; but I am not possibly actually identical to myself as both traveling to Patagonia and not traveling to Patagonia (in all my life). I.e., individuals are not undecided in the same way ISOAS may be undecided. However, they may be indirectly undecided with regard to the ISOAS in which they are given. This undecidedness which "affects" individuals cannot only be formulated (in our model), it is highly relevant for them. As mentioned, I am *not* concerned that there is some M where I am tortured to death tomorrow, but I am highly concerned if there is a successor to the actual M where I am tortured to death tomorrow. And this latter case is exactly formulated by: I am *possibly actually* identical to myself as being tortured to death tomorrow; or: I (actually) factually conditioned possibly will be tortured to death tomorrow. On the positive side, if such undecidedness "affects" individuals, they may (theoretically possibly), eventually, be able to *decide* something in the strong or "real" or actual sense: in the sense that they decide the course of their life.

If an ISOA is factually possibly identical with its successors, it is, of course, *actually* identical with its predecessors. My life today is identical to my life 50 years ago. It is the same life I lived then, and I am living now. Of course, we could declare this way of talking as merely informal. But within our model, we can make precise sense of it: if I am talking about my life, I am talking about an ISOA. However, with this, we get asymmetrical identity: an ISOA is identical to all its predecessors, but it is only possibly identical to its successors. Hence, the predecessors of an ISOA are only possibly identical to it. This may seem strange, but we must remember that we are not talking about identity in LS, i.e., about *logical* identity, anymore. We are talking about trans-logical factual identity, identity regarding facts. Mark that this asymmetrical identity only affects ISOAS, even in our model, individuals, SOAs (including Ms), and Ps are symmetrically identical to themselves. This may be seen as a reason to discard ISOAS. However, it is not a very good reason, at least not a logically valid one, since no inconsistency arises from it.

A good reason why we should accept ISOAS is that the *World* (W) in our normal understanding – different from PWs in PWS and from Ms in our model – is an ISOA: a consistently determined maximal ISOA. (Mark that the predecessors and the successors of a maximal ISOA also are maximal ISOAs, i.e., W always was and always will be a maximal ISOA.) Another reason that seems very good to me (but may seem bad to others) is that in such a world *and only in such a world* can we make decisions (in the full sense of the word). In a mere M, I do have actual possibilities. But since an M is only what it is and will never be anything else, these possibilities do not *turn* actual, and, hence, there is no decision as to which of them is turned actual – may that decision be that of a conscious subject or a "decision" (in the broader sense) by mere happenstance. Only if there is something that, as identical to itself, *goes on* from the actuality of M to the actuality of a successor of M is there such a thing as a decision.

There is one more problem to solve. In the past, W has, *in fact*, passed from one M to another. That is, there is not only continuity and succession regarding the past of W, but also dynamics: there is a factual "impulse" that has driven the world from older past stages to younger stages up to the present (note that, since this impulse is *factual*, it has, as such, no explanation whatsoever). But from the present onwards, W is not factually-identical to any MM but only possibly-identical to some Ms (that will have been). However, with this, there is no impulse for W to go on into the future since it *need not* become any of these possible MMs. That is, regarding the past, W is dynamical, but only *factually* dynamical, i.e., not in virtue of logical relations, since facts are without those. Hence, in *this* regard, W's past is only dynamic, not logical-mathematical structured. The future, on the other hand, is logically-mathematically structured, but it is not dynamic. There is, so to speak, no *need* for W to be dynamic in and into the future, since in LS all (consistent) possibilities can coexist, there is no need to select only one of them (as the actual). Because selecting one of the MMs that W is possibly-identical to as the actual W would be dynamic into the future: some possible MM turns actual, i.e., it turns out to be W.

Hence, to understand the world, we need to understand how the dynamic it factually has carries on into the future. An apparent solution would be that the past and the future of W overlap. However, this is unacceptable not only because the "region of overlap" would be inconsistently determined. The even more fundamental reason is that, with this, actuality would disappear, because according to our model, actuality is nothing else but the limit between the past and the future. If they do not limit one another, then there is no present. But the present is the only stage where SOAs and LS meet, where a P can express an SOA *directly*. On the other hand, a P can express an SOA *indirectly*, i.e., as a fact or future possibility, only by virtue of being part of a world where SOAs are expressed directly.

However, there is another solution. I cannot claim originality here since I cribbed this solution from Physics. But this is probably just for the best, for if a mere philosopher had come up with it, everybody would consider it fantastic speculation. So, the solution is: There *is* a limit between facticity and factually conditioned possibility, i.e., between the past and the future. But this limit is *unsharp*, or inexact, or uncertain – as in Heisenberg's "uncertainty principle". The reason why I do not want to call it "indetermined" is that this term suggests that there is no determination at all, i.e., that the limit between the past and the future is completely undefined. However, if, e.g., a photograph is unsharp, contours do not disappear completely, they just are not exact anymore. Hence, if we want to use the term "determined" in this context, we should say "underdetermined", or "restrictedly undetermined". I prefer the term "inexactness" – but that is just a question of terminology.

Of course, many people find it highly counterintuitive that the limit between the past and the future should be inexact. But what reason is there against it, other than our intuition that nothing should be inexact? Note that it is not an argument against the inexactness of the present that in LS everything is exact because the present is the limit between LS and the realm of facticity. The present is the "location" where LS and actuality coincide, i.e., LS extends, so to speak, from the present into the future, including the present itself. Nevertheless, with this, the present (though it participates in LS) still marks the limit of LS towards facticity. The limit between future and past is *also* the limit between present and past, i.e., the limit between LS and what is beyond LS. But *as such*, that limit cannot fall into LS, at least not completely. That which limits LS or "closes" it cannot lie within LS. This is visually-intuitively clear and has been confirmed by Kurt Gödel (1931) – or so we can interpret his theorem. Hence, the fact that all in LS is exactly determined cannot be an argument for the exactness of the limit of LS itself, because, for this, LS would have to contain its own limit – which is impossible.

If the present is inexact, then there are regions where past and future, i.e., facticity and factually conditioned possibility, are non-distinct (I use this term as different from "indistinct", which would apply to a simple overlap of past and future) or incompletely-distinct from one another, and hence, in this region, the realms of factual dynamics and logical-mathematical givenness of possibilities are non-distinct from one another. I.e., in this region, dynamics or *impulse* (through the realm of facticity) on the one hand, and possibility, i.e., *locatedness* in LS in the other, permeate or "perichorate" one another.

Note that the inexactness of the present may be very limited in time. A minimal inexactness is completely sufficient for the factual impulse of the world to carry on into the future. On the other hand, this minimal inexactness is also sufficient to "domesticate" this impulse. Because without the definition of factually conditioned possibilities, i.e., of possibilities with which the past-and-present is *compatible*, that impulse could go *anywhere*. However, for that requirement of compatibility to come to bear on that impulse, that impulse must permeate into LS, since only within LS as the space of logical relatedness can things be compatible or incompatible with one another. Hence, the inexactness of the present guarantees that, on the one hand, the world *does* go on, and that, on the other hand, it does not go on unrestrictedly anywhere, but that it goes on to actualize one of the MMs it is possibly-identical with.

If we now remember that, according to our model, the laws of nature should not be deterministic (in fact, they *cannot* be so, but I do not have the space to show this in this paper), there is always more than one MM W is possibly-identical with, regarding every present, i.e., regarding every transition from the past into the future. Hence, the present is always the location of *happenstance* (regarding W as a whole – partial transitions within W may very well be deterministic).

That is, with this, we have arrived at a (theoretically) *possible* explanation for *why* there is inexactness in the world. Mark that Heisenberg only tells us that there *is* inexactness (and how it fits into the physical world). But why on earth did God not only allow chance in the universe but also the inexactness of limits? Why did he add injury to insult to all orderly people? Because God is bound by logic. And logically-inevitably, if God accepted our model, they could not but center the World on happenstance.

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[i] Since this article is only a proposal, I will not make any references to other authors (with one exception). However, I do want to mention Guido Imaguire, to whom I am very grateful for many corrections and suggestions on the subject.

[ii] Cf. Utz 2024 on further details about factually conditioned necessity.